Lecture Notes for Math 251: ODE and PDE. Lecture 3: 1.3 Classifications of DEs

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1 Classifications of Differential Equations

Last Time: We solved some basic differential equations, discussed IVPs, and defined the general solution.

Now we want to classify two main types of differential equations.

Definition 1. If the unknown function depends on a single independent variable where only ordinary derivatives appear, it is said to be an **ordinary differential equation**. Example

$$y'(x) = xy \tag{1}$$

Definition 2. If the unknown function depends on several variables, and the derivatives are partial derivatives it is said to be a **partial differential equation**.

One can also have a **system of differential equations**

$$dx/dt = ax - \alpha xy \tag{2}$$
$$dy/dt = -cy + \gamma xy \tag{3}$$

Note: Questions from this section are common on exams.

Definition 3. The **order** of a differential equation is the order of the highest derivative that appears in the equation.

Ex 1: $y''' + 2e^t y'' + yy' = 0$ has order 3.
Ex 2: $y^{(4)} + (y')^2 + 4y''' = 0$ has order 4. Look at derivatives not powers.

Another way to classify equations is whether they are linear or nonlinear:

Definition 4. A differential equation is said to be **linear** if $F(t, y, y', y'', ..., y^{(n)}) = 0$ is a linear function in the variables $y, y', y'', ..., y^{(n)}$. i.e. none of the terms are raised to a power or inside a sin or cos.

Example 5. a) $y' + y = 2$
b) $y'' = 4y - 6$
c) $y^{(4)} + 3y' + \sin(t)y$

Definition 6. An equation which is not linear is **nonlinear**.

Example 7. a) $y' + t^4 y^2 = 0$
b) $y'' + \sin(y) = 0$
c) $y^{(4)} - \tan(y) + (y''')^3 = 0$

Example 8. $\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin(\theta) = 0.$

The above equation can be approximated by a linear equation if we let $\sin(\theta) = \theta$. This process is called **linearization**.

Definition 9. A **solution** of the ODE on the interval $\alpha < t < \beta$ is a function ϕ that satisfies

$$\phi^{(n)}(t) = f[t, \phi(t), ..., \phi^{(n-1)}(t)] \tag{4}$$

Common Questions:

1. (Existence) Does a solution exist? Not all Initial Value Problems (IVP) have solutions.

2. (Uniqueness) If a solution exists how many are there? There can be none, one or infinitely many solutions to an IVP.

3. How can we find the solution(s) if they exist? This is the key question in this course. We will develop many methods for solving differential equations the key will be to identify which method to use in which situation.

HW 1.3 # 1-6, 8, 12, 16, 22, 24, 27