Optimal State Estimation: Errata

Dan Simon

July 2, 2013

This is a list of errors in the book Optimal State Estimation, John Wiley & Sons, 2006. The main web site for the book is at `http://academic.csuohio.edu/simond/estimation`. My email address is listed on my home page at `http://academic.csuohio.edu/simond`.

I enthusiastically welcome feedback, comments, suggestions for improvements, and corrections. I also gratefully acknowledge Juan Luque, Ibrahim Abdel Hameed, Rick Rarick, Memo Ergezer, David Schwartz, Jeff Gove, Kevin Sharp, Stephan Busch, Yeoh WeeSoon, Michael Haralambous, Ville Kyrki, George Dontas, Felix Monasterio-Huelin, Cagdas Ozgenc, Cheng Zhong, Ning Lei, Roberto Rigamonti, Vincent Sircoulomb, Sami Fadali, Max Medvetsky, Jonathan How, Ismar Masic, Arthur Menikoff, Nedzad Arnautovic, Gabriel Zigelboim, Antje Westenberger, Laurent de Vito, Warner Losh, HunCheol Im, Martin Grossman, and James Tursa for pointing out many of the errata that are documented here.

1. For the probability distribution function, also know as the cumulative distribution function, the book should use the acronym CDF instead of PDF to avoid confusion with pdf (probability density function).

2. Use of the term "linear": Page 67 near the top refers to a "linear" transformation. Strictly speaking, this is not a linear but an affine transformation. There are two more references on the bottom of the page. Page 60 refers to a linear function (which is conventional usage). But then, to be consistent, the last sentence in the example should read "linear function" instead of "linear transformation."

3. Equation (1.11) should read

$$A^H = \begin{bmatrix} 1 & -4j \\ -2j & 5-j \\ 3+j & 1+3j \end{bmatrix}$$

4. In the fifth line after Equation (1.22) the word "invertible" should be changed to "non-invertible."

5. The discussion on pages 10 and 11 assumes that A is real. This can be corrected a couple of different ways. One way would be to explicitly state the assumption that A is real. Another way would be to allow A to be complex, in which case the word

"symmetric" should be changed to "hermitian" on the 3rd and 4th lines after Equation (1.31), and all of the tranpose symbols in the rest of Section 1.1.1 should be changed to hermitian transpose symbols.

6. The left side of Equation (1.32) should be $||x||_Q$.

7. When "singular values" are defined, it should be stated that they are positive. This clarification should be added to the text before (1.33), (6.36), and (11.113).

8. In Equation (1.45), the element in the first row and second column of $(A')^{-1}$ should be -1 instead of $+1$.

9. In the line above (1.49) on p. 14, A should be described as an $n \times n$ matrix.

10. Note that (1.54) appears to be inconsistent with (1.53). There is no standard definition for the derivative of a function with respect to a vector, but if we transposed the definition of (1.53), then we would run into inconsistencies in our later derivations, so we retain (1.53) and (1.54) as defined.

11. The lower right term in the matrix of (1.60) is missing a forward-slash.

12. The second equation in Equation (1.62) on p. 16 should state that the partial of $x^T A^T$ is equal to A^T.

13. t terms are missing from the second matrix in Equation (1.73).

14. On page 20, the statement that begins "Another interesting fact ..." is incorrect.

15. Following Equation (1.82) add the sentence, "See any linear systems text [Bay99, Che99, Kai80] for the computation of $e^{\hat{A}t}$.

16. In (1.87) on p. 23, $\tilde{x}_2$ should be changed to $\tilde{x}_n$.

17. On page 26 just before Equation (1.99), $\sin x_4$ and $\cos x_4$ should be replaced with $\sin \bar{x}_4$ and $\cos \bar{x}_4$.

18. Equation (1.102) on page 27 is the first place we introduce a discrete-time system. I use F_{k-1} and u_{k-1} on the right side of the equation, while some other authors use F_k and u_k. This is not a mistake, but the reader should be aware that there is no standard notation, and I choose the notation that I use as a matter of personal preference.

19. The upper summation limit in Equation (1.108) should be $L - 1$. The two upper summation limits in Equation (1.109) should be $n - 1$ and n respectively. The two upper summation limits in Equation (1.112) should be $n - 1$.

20. The caption of Figure 1.2 should say that $x(0.5) \approx x(0) + A_1$, $x(1) \approx x(0.5) + A_2$, etc. A similar correction needs to be made to the caption of Figure 1.3.

21. All of the A and $\hat{A}$ terms from Equation (1.135) through Theorem 8, and also in Equation (1.142), should be changed to F and $\hat{F}$ respectively.

22. In the paragraph at the top of page 41 the word "controllability" should be changed to "observability" (twice).

23. Before Equation (1.166) note that $\bar{x}$ is defined to be equal to $M^{-1}x$. The u is missing from the end of the first expression in Equation (1.166).

24. An x is missing from the last line of Equation (1.167).

25. Remove the u term from the y expression in Equations (1.167) and (1.168).

26. Exercise 1.11 is stated wrong. It should read: Show that $|e^{At}| = e^{Tr(A)t}$ for any square matrix A.

27. The equation that accompanies Exercise 1.13 should read

$$p = p_0 + \dot{p}_o t + \frac{1}{2}\ddot{p}_0 t^2$$

28. In Exercise 1.19(d), $\cos(t)$ should be $|\cos(t)|$ (otherwise the propellent flow rate could be negative).

29. The left side of the last equation in Equation (2.14) should have X instead of x.

30. In (2.17) on p. 54, the term on the right side of the first equation should be $f_{X_1}(x_1|X_2 = x_2)$.

31. The last line of Equation (2.43) should have the absolute value of a.

32. The first line of Equation (2.51) should have $f(z_1, z_2)$.

33. Equation (2.55) on p. 62 in missing a right parenthesis.

34. The sentence following Equation (2.56) should read, "The correlation coefficient is a normalized measurement of the correlation between two RVs X and Y."

35. Equations (2.62) and (2.63) can be derived even if x and y are not independent.

36. Example 2.11 on page 65 is partly wrong: $E(I_1) = -1/4$ and $E(I_2) = 1/4$. All $V > 0$ give $I_1 = 0$, which gives an I_1 pdf which has an impulse of area $1/2$ at $I_1 = 0$. All $V < 0$ give $I_1 = V$, which gives an I_1 pdf which is uniform from -1 to 0 with value $1/2$. Therefore, $E(I_1) = (1/2)(0) + (1/2)(-1/2) = -1/4$. A similar derivation holds for I_2. This also means that $E(I_1)E(I_2) = -1/16$ at the end of the example.

37. The left side of Equation (2.76) should be Y instead of y.

38. The second sentence of the last paragraph on page 70 has one too many "only"s.

39. Exercise 2.13 should read, "... The pdf's of X and V ..." instead of "... The pdf's of X and B ...".

40. The top figure of Figure 2.5 should show pdf(x) equal to 0.5 instead of 1 (so that is has an area of 1).

41. On page 81, two lines below Equation (3.7), "greater than" should be changed to "greater than or equal to".

42. The right side of Equation (3.15) should be multiplied by 2.

43. On page 84 the second paragraph of Section 3.3 should read, "In this section, we show how to recursively compute the weighted least squares estimate of a contant *vector.*"

44. On page 84, "estimation error mean" just before Equation 3.21 should be hyphenated to be consistent with "error-estimation covariance" on page 85.

45. The term "zero-mean" after Equation (3.21) should instead be "zero mean."

46. The first line of Equation (3.22) has a couple of extra right parentheses, $\hat{x}_1$ should be changed to $\hat{x}_{1k}$, and $\hat{x}_n$ should be changed to $\hat{x}_{nk}$. Also it should be noted that the cost function in this section is different than the cost function in the previous section, and we are summing quantities that may have different units.

47. Page 84, 6th line from the bottom, should read "...equal to x for all k..." instead of "equal to x_k for all k."

48. The ϵ in Equation (3.24) is missing a transpose.

49. At the bottom of page 85, the phrase "it will also be consistently close to zero" should be changed to "it will also be consistently as close as possible to zero."

50. On page 89 following Equation (3.49), $P(0)$ should be changed to P_0 (twice).

51. On page 92, the $P(2,2)$ legend in Figure 3.1(b) is missing a parenthesis.

52. On page 96, the footnote should read "...present and *past* inputs..."

53. Exercises 3.4(b) and 3.5(b) are missing the word "the."

54. Exercise 3.4(c) should read, "... what is the variance of the voltage *estimation error?*"

55. The equation in Exercise 3.8(d) should read

$$\hat{x} = \text{argmin}_y \max_x |x - y|$$

56. Q_k should be replaced with $\tilde{Q}_k$ in Equation (4.14).

57. Note that Equation (4.18) only holds if A is constant between sample times [Bay99, pp. 252ff.].

58. The footnote at the bottom of page 111 should refer to zero-order holds rather than first-order holds.

59. Problem 4.13(b) does not make sense.

60. Item 3(b) on page 124 is wrong.

61. The first paragraph on page 126 and the last paragraph on page 127 refer to P as the covariance of the estimate. It should instead refer to P as the covariance of the estimation *error*. This mistake may be repeated elsewhere in the text also.

62. Bullet 3 on page 140 could be implemented by using the Joseph form of the covariance update of Equation (5.19). But there may be computational reasons that the less stable but computationally cheaper covariance update is used, and so the user may want to symmetrize P every once in a while.

63. Exercise 5.9(a) (page 147) should read "...$P_0^+ = 1$..." instead of "...$E(x_0^2) = 1$..."

64. In the third line of Equation (6.22), y_{33} should be changed to y_{31}.

65. On page 157 on the line after Equation (6.34), the word "and" should be changed to "an".

66. The text following Equation (6.36) says that $P^T P$ is symmetric, and the eigenvalues of a symmetric matrix are always real and nonnegative. Instead of "symmetric" it should say "symmetric positive definite."

67. The text following Equation (6.38) says that positive definiteness is equivalent to invertibility, but it should instead say that positive definiteness implies invertibility.

68. Equation (6.111) does not make sense as written – instead of the equal sign we should use an assignment expression, or use a different symbol on the left and right sides of the equation.

69. The U-D composition in Section 6.4 on page 174 could be obtained by writing $U = SE^{-1}$ and $D = E^2$, where S is obtained from the Cholesky decomposition of P on page 160, and E is a diagonal matrix with $E_{ii} = 1/S_{ii}$.

70. In Equation (7.1), $E[w_k v_j^T] = M_k \delta_{k-j+1}$ should be changed to $E[w_{k-1} v_j^T] = M_k \delta_{k-j}$ (same thing in Equation (7.12)).

71. Section 7.2.1 should be discussed in the context of LTV systems, not LTI systems.

72. The $[\ 1 \ \ 1\ ]^T$ array in Equation (7.46) should be changed to $[\ 0 \ \ 1\ ]^T$.

73. In the 2nd expression in Equation (7.34), the last $\hat{x}_k^+$ should instead be $\hat{x}_k^-$. This changes the experimental results in Table 7.2 so that the Measurement Differencing column becomes identical to the Augmented Filter column.

74. In the P_{k+1}^- expression in Equation (7.34), the last M_k should be changed to M_k^T, and the last M_k^T should be changed to M_k.

75. P_0 in the second sentence after Equation (7.43) should be P_0^+.

76. Equation (7.59) should read
$$\hat{x}_k^+ = \frac{1}{2}\hat{x}_{k-1}^+ + \frac{3}{4}y_k$$

77. The 3rd and 4th expressions of Equation (7.69) should read
$$P_{11}^- = \frac{K_1 R}{1 - K_1}$$
$$P_{12}^- = \frac{K_2 R}{1 - K_1}$$

78. Equation (7.70) should read
$$\lambda = \frac{\sigma_w T^2}{\sqrt{R}}$$

79. Note that the derivation of Section 7.3.3 is essentially the same as that in Section 5.4.1

80. The discussion of Equations (7.108) and (7.109) has some conceptual errors and needs to be rewritten. The discussion starting at Equation (7.110) is fine, but the material leading up to it is wrong.

81. On page 212, on the 4th line before Section 7.5.1, "this will presented" should be replaced by "this will be presented."

82. Equation (7.158) should read
$$\rho_k = e_k^T - \sum_{i=1}^{k-1}(\rho_i e_k)\rho_i$$

Equation (7.160) should read
$$\rho_k = e_1^T - \sum_{i=1}^{k-1}(\rho_i e_1)\rho_i$$

83. In Equation (7.165) on page 220, $d_i(k)$ should be changed to d_{ki}.

84. In Equation (8.18), $v(\tau)$ should be replaced with $v^T(\tau)$.

85. In the first two lines of Equation (8.20), w_k should be replaced with w_{k-1}.

86. In Equation (8.22), Λ should be deleted. In Equation (8.23) the definition of Λ should be deleted, and the N in $v_k \sim N(0, R)$ should be deleted.

87. In Equation (8.94) on page 245,
$$\begin{bmatrix} S_{11}^T \\ S_{21}^T \\ S_{31}^T \end{bmatrix}$$
in the second line should be changed to $\begin{bmatrix} S_{11}^T & S_{21}^T & S_{31}^T \end{bmatrix}$. A similar change needs to be made to the fourth line of the equation.

88. There are several places in Section 8.4.2 where C should be changed to $\tilde{C}$: The K expression in Equation (8.118), Equation (8.125), the $\dot{K}$ expression in Equation (8.126), and Equation (8.127).

89. On page 252, three lines below Equation (8.129), "sdomain" should be changed to "s domain" (with a space between the s and "domain").

90. In the first sentence in the second paragraph on p. 260 there is one too many "the"s.

91. There is not exactly a "mistake" here, but the notation in Section 9.2 should be changed to be more intuitive, so that y_{k-1} is replaced with y_k on the right side of (9.11). This change in notation will rippple through the entire section.

92. Equation (9.53) should be $\hat{x}_{k+1-i,k} = \hat{x}_{k+1-i,k-1} + L_{k,i}(y_k - H_k x_k^-)$.

93. In the line following Equation (9.57), x_f should be $\hat{x}_f$, and x_b should be $\hat{x}_b$.

94. The 2nd sentence before Example 10.1 on page 303 should read, "For example, $\max(1 - x^2) = 1$ because the maximum of $(1 - x^2)$ is 1, but $\mathrm{argmax}_x(1 - x^2) = 0$ because $(1 - x^2)$ attains its maximum value when $x = 0$."

95. The fourth line after Equation (10.24) needs a space before the word "we."

96. In Equation (10.35) the two Λ terms should be deleted. The last expression in Equation (10.37) should read $Q = BQ_cB^TT$.

97. The text prior to Equation (10.73) says that it is a Riccati equation, but it is actually a Lyapunov equation.

98. On page 315 in the paragraph below Equation (10.79), "process noise" should be changed to "measurement noise," and vice versa.

99. On page 315 in the line above Equation (10.82) a right parenthesis is missing.

100. On page 316 in the second paragraph, the word "measurement" goes outside the page boundaries.

101. On page 316, the paragraph following Equation (10.86) indicates that the derivative of J with respect to K must be computed numerically, but it can probably be computed analytically.

102. On page 321 in Equation (10.107), $\hat{x}^-(k)$ should be changed to $\hat{x}(k)$.

103. On page 321 in Equation (10.112), there should be a comma after $x(k)$ in the first two expressions, and the tranpose symbols should be dropped. In the first expression, the right parenthesis before $H(k)$ should be a right bracket.

104. In the second equation of Equation (11.1) a subscript k is missing.

105. The text before Equation (11.2) says, "As derived in Section 5.1, the Kalman filter equations are given as follows:" – but in fact the one-step Kalman equation in Equation (11.2) was first introduced in Section 5.3.

106. In the fourth line of Equation (11.6) and the second line of Equation (11.7) there is a superscript minus sign missing from one of the P_k terms.

107. On page 343 in Equation (11.36), P_0, Q_k, R_k, and S_k are used without any explanation. Note that P_0, Q_k, and R_k are not necessarily the same as in the Kalman filter. The terms are explained at the top of page 344.

108. In the last paragraph on page 343, the word "pdf" should be changed to "statistics" (three times).

109. Equation (11.25) should specify that x_0 is constant but unknown. This gives the additional term $\lambda_0^T(x_0 - k_0)$ in Equation (11.27), which in turn affects the following three equations, and makes the final result of Equation (11.33) correct as written.

110. In Equation (11.38), < 1 should be changed to < 0.

111. The line prior to Equation (11.47) contains the phrase "(with respect to x_0 and w_k)". That phrase should be deleted.

112. The phrase "to the cost function" should be replaced with "from the cost function" prior to Equation (11.73).

113. In the second line of Equation (11.79) the pre and post P_k^{-1} terms can be omitted to make the derivation simpler. There's a P_k^{-1} term at the beginning of the equation and also a P_k^{-1} term at the end of the equation – they can both be omitted.

114. In Equation (11.93) K_k should be K.

115. A right parenthesis is missing from footnote 2 on page 355.

116. In Equation (12.2) the first term in the denominator should be removed.

117. On page 378 in Equation (12.21), S_1 and S_2 need to be defined. They are the weighting matrices in Equation (12.18).

118. On page 383 in Equation (12.40), the subscripts of the terms on the left of two of the equations should be $k + 1$ instead of k.

119. In the first paragraph of Section 13.1, "are" should be changed to "is".

120. It would make more sense to switch the order of Sections 13.2.2 and 13.2.3.

121. $h_k(\hat{x}_k^-, \cdot, t_k)$ should be changed to $h_k(\hat{x}_k^-, \cdot)$ in Equations (13.29) and (13.33).

122. In the line following Equation (13.32), F should be changed to A.

123. In Equation (13.37), 1/2000 should be changed to 2000, 1/2500 should be changed to 2500, and 1/250000 should be changed to 250000.

124. The plus superscripts on the right side of Equation (13.52) should both be changed to minus superscripts.

125. In Hybrid2.m, which was used to produce the Example 13.3 results, I mistakenly use H in both the continuous-time propagation and in the discrete-time update. The problem is in both the first-order EKF and in the iterated EKF, but not in the second-order EKF.

126. Equation (14.7) should read $\bar{y}_1 = 0$, and (14.9) should read $\bar{y}_2 = E(\cos \tilde{\theta})$. The equations as written in the book are correct but confusing. It could also be clearer if it is stated before (14.7) and (14.9) that $\tilde{r}$ and $\tilde{\theta}$ are zero mean.

127. y_u should be changed to $\bar{y}_u$ in Section 14.2.2.

128. In Equation (14.59) u_k and t_k should be replaced with u_{k-1} and t_{k-1}.

129. Check the numbers in Table 14.1 on page 457 using HybridSimplex.m – I have some questions if the Simplex results are correct.

130. Just before Equation (15.20), y_k should be changed to Y_k. The same change should also be made in Equation (15.21), in the text before Equation (15.21), and in step (3e) on page 468. Similarly, y_{k-1} should be changed to Y_{k-1} twice in the first paragraph of Section 15.3.1.

131. In the [Bat82] reference on page 503, the quotation mark is incorrect.

132. On p. 507 [Gre01] should include "2nd edition" to be more precise.

133. On p. 516 [Sea67] is out of order.