Abstract—A DC motor is mounted to a Mauch SNS prosthetic knee to obtain an active prosthetic knee. Evolutionary optimization and derivative-based optimization are used to identify system parameters, and to tune a proportional-integral-derivative (PID) controller for knee ankle tracking during swing phase. A Kalman filter is used to estimate knee angle velocity on the basis of the measured knee angle for feedback to the controller. The performance of the optimization algorithms are evaluated based on integral square error (ISE) between experiment and simulation for the system identification problem, and tracking ISE for the control problem. Results show that for system identification, particle swarm optimization (PSO) gives better results than sequential quadratic programming (SQP) and biogeography-based optimization (BBO). Then PID controller optimization is performed while considering nine different shank lengths. BBO achieves the best average overall ISE, and PSO shows the fastest convergence. Finally, we see that increasing shank length results in an increase in the optimal proportional gain of the controller and a decrease in the optimal derivative and integral gains.

I. INTRODUCTION

Recent decades have seen significant advances in limb prosthetics. However, there is still a large gap between state-of-the-art prosthesis performance and able-bodied walking [1]. Lower limb amputation is the most frequent amputation, and transfemoral (above knee) amputation is the most frequent type of lower limb amputation.

Passive Prostheses – Most knee prostheses are passive, which means that they do not include electronic controls [2]. Prostheses consist of a foot, shank, knee, and a socket for interfacing with the amputee’s residual limb. The artificial knee provides flexion-extension for the prosthetic leg. The knee is connected to the shank, which forms the internal frame or skeleton of the prosthetic limb. The shank provides structural support and is typically composed of metal rods. Though most prosthetic limbs have these basic components, each one is unique and is designed for a specific type of amputation and user activity. Passive prostheses are comprised of springs and dampers that are configured to prevent stumbling and to emulate able-bodied gait.

Semi-active Prostheses – Microprocessor prosthetic leg technology is relatively recent (within the last couple of decades) and provides users with more sophisticated control than passive prosthetics. These types of prostheses are referred to as semi-active. Microprocessor prosthetic knees are designed to help the amputee walk with a gait that is more stable and efficient, and that resembles a natural gait more than passive prostheses. Semi-active prostheses are especially attractive to active amputees, who require better performance than that provided by passive prostheses [3].

Active Prostheses – For even better performance than semi-active prostheses, we can use motors in the knee or ankle. These types of prostheses are called active prostheses. There are only a few commercially available active prostheses, including the transfemoral Power Knee [4] and the transtibial BIOM T2 [29]. Other active prostheses are currently in development at various research labs [5].

The main goal in this paper is to design, optimize, and implement PID control for various shank lengths in active knee prostheses. We use a Mauch SNS, which is a widely-used passive prosthesis [6-8], and attach it to an EMG-30 geared DC motor. We modified the Mauch SNS by removing the damper and driving it instead with a DC motor.

Our work is preliminary because the EMG-30 is not sufficiently powerful for a prosthetic knee during stance. Our work provides a conceptual approach for system identification and control optimization of active prostheses during swing phase. In order to obtain adequate performance during stance, we will need a more powerful motor or a ball screw. After we upgrade the mechanical configuration, we will be able to use the same methods reported here for system identification and controller optimization on a system that is closer to commercial viability.

First Research Objective - The first objective of this research focuses on modeling the EMG-30 DC motor and the motor / shank combination. System identification is performed by estimating the unknown parameters using biogeography-based optimization (BBO) [9-10] and particle swarm optimization (PSO) [11-12], which belong to the evolutionary algorithm (EA) family, and sequential quadratic programming optimization (SQP), which is a gradient-based optimization algorithm [13].

EAs were originally motivated by natural selection, which has produced fit behaviors and configurations in biological species. EAs are well-suited for large and complex optimization problems. A population of candidate solutions evolves to produce candidate solutions that improve over
The evolution of the population is regulated by operators that are motivated by natural processes, such as natural selection, insect swarming, species migration, etc. The candidate solutions are assessed with a given objective function. The better solutions have a higher probability to share their features with other candidate solutions, and thus improve the population in later generations [14]. We investigate three different algorithms for system identification and controller optimization. Our approach is motivated by the fact that each optimization algorithm has its own advantages; some perform better on certain problems, while others perform better on other problems. We choose to use BBO and PSO in this paper because they are both easy-to-implement algorithms which have shown considerable success in previous research. BBO is chosen as a representative EA in which candidate solution directly shares information with other candidate solutions [10]. PSO is chosen as a representative EA in which the optimization process is based more on swarm behavior rather than direct information sharing [15]. We also use sequential quadratic programming (SQP), which is a numerical method for constrained nonlinear optimization problems. SQP divides an optimization problem into a sequence of sub-problems, each of which is a quadratic approximation with linearized constraints. For unconstrained problems SQP reduces to Newton’s method. We consider SQP as an alternative to EAs such as BBO and PSO.

**Second Research Objective** - The second objective of this research is to optimize the motor controller parameters. The center of mass of the Mauch knee and shank combination changes with shank length. Consequently, the optimal PID controller parameters are functions of shank length. For each variation in shank length, a PID controller is tuned using BBO, PSO, and SQP.

Section II provides an overview of the methods in this paper, including system identification, validation, and controller optimization. Section III discusses the system model. Section IV discusses the prosthetic hardware and the experimental test. Section V presents results. Section VI presents a conclusion and a discussion of future work.

## II. METHODOLOGY

In this section we explain the techniques used to identify the motor parameters and to tune the PID controller parameters off-line. For a given reference input, the system response is recorded, and then the outputs are used to identify the system parameters using PSO, BBO, and SQP. Finally, within the simulation environment, the optimization algorithms are used to tune the controller.

### A. Parameter Identification

We use particle swarm optimization (PSO) [11]-[12], [16]-[17], biogeography-based optimization (BBO) [10], and sequential quadratic programming (SQP) [13] to identify the parameters of the motor and motor / shank combination. The quality of the estimated parameters is determined by the integral square error (ISE) of the difference between the estimated output of the experimental system (angular velocity), and the output of the simulation model. Fig. 1 outlines the system identification process.

![Figure 1. Parameter identification approach](image)

### B. Direct Measurements

In order to reduce the number of parameters to be found by the identification process, experimental tests are performed to directly measure some of the motor parameters. The experimental tests are described as follows.

1) Resistance $R_m$ and inductance $L_m$ – We directly measure motor resistance and inductance.

2) Motor speed constant $K_m$ – We connect the shaft of the EMG-30 DC motor to a second motor with a mechanical coupling. We use the second motor to drive the EMG-30 DC motor, and then we measure the voltage induced across the EMG-30 motor terminals. Dividing the terminal voltage by the shaft velocity gives us the motor speed constant.

### C. Controller tuning for different shank lengths

We vary the shank length and re-optimize the controller for each shank length to find the optimal PID gains for various shank lengths. Fig. 2 depicts the off-line controller tuning technique. The optimization algorithm uses the estimated knee velocity from the system model to calculate ISE.

![Figure 2. Off-line tuning for PID control. The KF block is a Kalman filter.](image)

## III. SYSTEM MODELING

The EMG-30 DC motor [19] is modeled by representing the armature as a circuit with resistance $R_m$ connected in series with a coil with inductance $L_m$ and supplied by a voltage source $e_s(t)$. The shank is connected to the DC motor...
and is considered as a radial load due to gravity. The motor variables and parameters are defined in Table I, and a schematic of the motor-prosthesis combination is shown in Fig. 3.

### TABLE I. MOTOR VARIABLES AND PARAMETERS

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>$i_s(t)$</td>
<td>Armature Current</td>
<td></td>
</tr>
<tr>
<td>$I_m$</td>
<td>Armature Inductance</td>
<td></td>
</tr>
<tr>
<td>$R_m$</td>
<td>Armature Resistance</td>
<td></td>
</tr>
<tr>
<td>$e_m(t)$</td>
<td>Back-emf</td>
<td></td>
</tr>
<tr>
<td>$K_b$</td>
<td>Back-emf Constant</td>
<td></td>
</tr>
<tr>
<td>$T_L(t)$</td>
<td>Load Torque</td>
<td></td>
</tr>
<tr>
<td>$\omega_m(t)$</td>
<td>Rotor Angular Velocity</td>
<td></td>
</tr>
<tr>
<td>$T_m(t)$</td>
<td>Motor Torque</td>
<td></td>
</tr>
<tr>
<td>$J_m$</td>
<td>Rotor Inertia</td>
<td></td>
</tr>
<tr>
<td>$\theta_L(t)$</td>
<td>Rotor Displacement</td>
<td></td>
</tr>
<tr>
<td>$B_m$</td>
<td>Viscous Friction Coefficient</td>
<td></td>
</tr>
<tr>
<td>$K_T$</td>
<td>Torque Constant</td>
<td>N</td>
</tr>
<tr>
<td>$m$</td>
<td>Prosthetic Knee Mass</td>
<td></td>
</tr>
<tr>
<td>$L_s$</td>
<td>Distance to Center of Mass</td>
<td></td>
</tr>
</tbody>
</table>

Starting with the control input voltage $e_f(t)$, the equations for the motor circuit are given as follows:

$$T_m(t) = K_i i_s(t)$$  \hspace{1cm} (1)

Back-emf voltage is represented as follows:

$$e_m(t) = K_b \frac{d\theta_m(t)}{dt} = K_b \omega_m(t)$$  \hspace{1cm} (2)

We use Kirchhoff’s voltage law (KVL) to derive

$$\frac{di}{dt} = \frac{1}{L_m} e_s(t) - \frac{R_m}{L_m} i_s(t) - \frac{1}{L_m} e_m(t)$$  \hspace{1cm} (3)

$$\frac{d^2\theta_m(t)}{dt^2} = \frac{1}{J_m} T_m(t) - \frac{1}{J_m} T_L(t) - \frac{\theta_m}{J_m} \frac{d\theta_m(t)}{dt}$$  \hspace{1cm} (4)

where $T_L(t)$ represents the motor’s load torque:

$$T_L = \frac{mgL_s}{n} \sin(\theta_L)$$  \hspace{1cm} (5)

From the above, motor velocity is a function of input voltage and motor load, which in this case is shank weight:

$$\Omega_m(s) = \frac{K_i E_a(s)}{L_m J_m s^2 + (R_d J_m + B_m L_m) s + (K_b K_i + R_m B_m)}$$

$$- \frac{(J_m s + B_m) T_L(s)}{1 + (R_m + L_m) s (J_m s + B_m)}$$  \hspace{1cm} (6)

The only measurable parameters are armature resistance and inductance. So the unknown parameters are $J_m$, $B_m$, $K_b$, and $K_i$. When considering the combination of the shank and the Mauch knee, the unknown parameters include the total moment of inertia $J_k$ and viscous friction $B_k$.

### IV. EXPERIMENTAL SYSTEM OVERVIEW

The prosthetic knee consists of a Mauch SNS knee, minus its hydraulic control cylinder, attached to an EMG-30 DC motor. A metal tube comprising the shank is attached to the Mauch knee, and a standard plastic SACH (solid ankle, cushioned heel) foot is connected to the end of the shank. Fig. 3 shows the active prosthetic knee schematic. The EMG-30 has a rated speed of 170 rpm, rated current of 530 mA, no-load speed of 216 rpm, no-load current of 150 mA, stall current of 2.5 A, and rated power output of 4.22 W.

![Mauch knee equipped with DC motor and encoder.](image)

As depicted in Fig. 4, the encoder measures angular knee position. Estimating angular velocity is essential for PID control. Noise and discretization result in errors in measured angular position, which are magnified in the velocity calculation. Noise amplification of finite-difference calculations is one of the main problems with numerical differentiation [23]. For these reasons we use a Kalman filter to estimate the motor’s angular velocity.

**The Kalman filter** can estimate the states of a dynamic system, and is commonly used to estimate velocity [24], [25]. Suppose we have the following state equations:

$$x_{k+1} = Ax_k + Bu_k + w_k$$  \hspace{1cm} (7)

$$y_k = Cx_k + z_k$$  \hspace{1cm} (8)

where $x$ is the state; $y$ is the measured output; and $w_k$ and $z_k$ are process noise and measurement noise respectively. Defining the state vector components as position and velocity, and the measurement as the encoder based position output, we write linear system equations as follows:

$$x_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0.5T^2 \\ T \end{bmatrix} u_k + w_k$$  \hspace{1cm} (9)

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + z_k$$  \hspace{1cm} (10)

where $T$ is the discretization step size and $u_k$ is a known input. The Kalman filter estimates the state as follows:

$$K_k = AP_k C^T (CP_k C^T + S_k)^{-1}$$  \hspace{1cm} (11)

$$\hat{x}_{k+1} = (A\hat{x}_k + B u_k) + K_k (y_{k+1} - C \hat{x}_k)$$  \hspace{1cm} (12)
\[ P_{k+1} = A P_k A^T + S_w - A P_k C^T S_z^{-1} C P_k A^T \]  
\[ (13) \]

where \( K_k \) is the Kalman gain matrix and \( P_k \) is the estimation error covariance. Tuning parameters \( S_z \) and \( S_w \) are the measurement and process noise covariances, respectively. As an ad-hoc modification to the filter, we reset the filter estimate and covariance whenever the measurement residual exceeds a certain threshold. This allows the velocity estimate to re-initialize when there is a step change in the acceleration.

V. RESULTS

By trial and error we set the filter covariances as follows:

\[ S_z = 3, \quad S_w = \begin{bmatrix} 10^{-8} & 10^{-6} \\ 10^{-6} & 10^{-8} \end{bmatrix} \]

Fig. 5 depicts measured experimental angular position versus estimated angular position for the EMG motor when excited with a ramp input voltage. Angular velocity is satisfactorily estimated by the Kalman filter.

![Motor estimated position vs. measured position](image)

Figure 5. Motor estimated position vs. measured position.

Hardware setup and implementation – We compared optimization algorithms by executing each one for the same number of function evaluations. The values for all parameters have search ranges from \( 10^{-6} \) to \( 10^{-1} \), numbers that were obtained by trial and error.

A chirp signal is applied to the EMG-30 motor. The estimated output angular velocity is used to identify model parameters and optimize controller parameters.

A chirp signal with a 10 V amplitude and frequency range 1–10 Hz is used to obtain motor responses and to identify model parameters off-line. The chirp signal frequency range was selected to be near the dominant frequencies of the human gait cycle [26]. As described earlier, we measured both the resistance and inductance of the EMG-30 motor and obtained \( L_m = 230 \) mH and \( R_m = 15.3 \) \( \Omega \). To find the motor constant we conducted the experiment described in Section II. The ratio of induced voltage to angular velocity was found to be \( K_B = 0.0115 \) V/sec.rad\(^{-1}\).

The remainder of this section discusses the results of DC motor identification, shank identification, and PID controller optimization for various shank lengths.

A. DC Motor Parameter Identification

The recorded response of an unloaded EMG-30 motor is illustrated in Fig. 5. Since our primary goal here is to identify the parameters of the EMG-30 motor model, the input-output response is used to find the parameters that make a simulation model match the experimental input-output response.

The maximum number of SQP function evaluations was limited to 6000, and the parameter estimates and function tolerance were set to \( 10^{-4} \).

In order to see how sensitive BBO and PSO performance are, a sensitivity analysis test is carried out for each algorithm. We identify each algorithm’s sensitivity to each of its parameters by three different levels: “low” means less than 10% variation from the best solution, “medium” means variation between 10% and 25%, and “high” means more than 25% variation from the best solution. The results of the parameter sensitivity tests are shown in Table II for BBO, and in Table III for PSO. The “step” column in each table shows the increment that we used to test each parameter.

<table>
<thead>
<tr>
<th>Table II. BBO Algorithm Parameter Sensitivity Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>Number of generations</td>
</tr>
<tr>
<td>Population size</td>
</tr>
<tr>
<td>Mutation probability</td>
</tr>
<tr>
<td>Number of elites</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table III. PSO Algorithm Parameter Sensitivity Test</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>Number of generations</td>
</tr>
<tr>
<td>Population size</td>
</tr>
<tr>
<td>Correction factor</td>
</tr>
<tr>
<td>Acceleration constant</td>
</tr>
<tr>
<td>Cognitive parameter</td>
</tr>
<tr>
<td>Social parameter</td>
</tr>
</tbody>
</table>

From Table II the best BBO parameter set is as follows: 100 generations, 60 candidate solutions per generation, 10% mutation probability per independent variable per candidate solution per generation, and 2 elite individuals per generation, which means that the best 2 individuals each generation replace the 2 worst individuals in the following generation.

From Table III the best set of PSO parameters are as follows: 100 generations, 60 candidate solutions per generation, correction factor = 2, acceleration constant = 0.5, cognitive parameter = 0.1, and social parameter = 0.3.

Table IV depicts the optimized parameter values using different optimization methods. The values are obtained by running each algorithm 20 times because of the stochastic nature of the algorithms. We are confident about these results since it is clear from Table IV that \( K_B \approx K_I \) which is expected for permanent magnet DC motors. \( J_m \) is the total inertia of the motor and gears reflected to the output shaft. This includes
a term $n^2 J_m$, which explains why $J_m$ is so high. The rotor alone would have a much smaller inertia (on the order of $10^{-6}$ kg·m²).

We conducted t-tests [27] on our results and found that the improvement of PSO is statistically significant compared to both SQP and BBO, with a 95% confidence level. However, the performance of SQP and BBO are similar enough that their differences are not statistically significant.

### Table IV. Estimated DC Motor Parameters and ISE (Average +/- Standard Deviation over 20 Simulations)

<table>
<thead>
<tr>
<th></th>
<th>$K_i$ [N.m.A⁻¹]</th>
<th>$K_v$ [V.sec.rad⁻¹]</th>
<th>$J_m$ [Kg.m²]</th>
<th>$B_m$ [N.m.sec]</th>
<th>ISE [rad.sec⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQP</td>
<td>0.0513 ± 0.0419</td>
<td>0.0507 ± 0.0433</td>
<td>0.0052 ± 0.0045</td>
<td>0.0046 ± 0.004</td>
<td>11.49 ± 9.01</td>
</tr>
<tr>
<td>BBO</td>
<td>0.0450 ± 0.0295</td>
<td>0.0401 ± 0.0355</td>
<td>0.0043 ± 0.0045</td>
<td>0.0058 ± 0.004</td>
<td>10.60 ± 8.98</td>
</tr>
<tr>
<td>PSO</td>
<td>0.0502 ± 0.0367</td>
<td>0.0323 ± 0.0373</td>
<td>0.0060 ± 0.0042</td>
<td>0.0043 ± 0.004</td>
<td>7.450 ± 9.53</td>
</tr>
</tbody>
</table>

Although PSO performs best on average, out of the 20 simulation runs the lowest ISE value of 5.28 rad/sec was found by SQP (not shown in the table). Fig. 6 shows that, on average, PSO converges faster than BBO and SQP.

![Average convergence of ISE for different algorithms during system identification](image)

Although the optimization algorithms found parameter values that gave good ISE values, other reference signals may not necessarily result in low ISE values. To test this possibility we applied different reference signals and compared the motor model and the estimated velocity from the experimental system. Results (not shown here) confirm that even with different reference inputs, the identified model still matches the experimental model well.

### B. Shank Parameter Identification

The previous procedure of motor parameter identification is repeated with the motor-shank combination to estimate the combined moment of inertia $J_k$ and the viscous friction coefficient $B_k$. Optimization results are shown in Table V.

Although the performance of all three optimization algorithms in Table V are similar, PSO gives the best performance. BBO has the smallest ISE standard deviation, which shows that it performs most consistently. Table V shows that the identified moment of inertia is about the same for all three algorithms.

### Table V. Estimated Shank Parameters and ISE (Average +/- Standard Deviation over 20 Simulations)

<table>
<thead>
<tr>
<th></th>
<th>$A_k$ [Kg.m]</th>
<th>$B_k$ [N.m.sec]</th>
<th>$I_SE$ [rad.sec⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQP</td>
<td>0.0158 ± 0.0041</td>
<td>0.0054 ± 0.0037</td>
<td>13.27 ± 7.2400</td>
</tr>
<tr>
<td>BBO</td>
<td>0.0165 ± 0.0041</td>
<td>0.0012 ± 0.0037</td>
<td>10.78 ± 6.5550</td>
</tr>
<tr>
<td>PSO</td>
<td>0.0156 ± 0.0039</td>
<td>0.0066 ± 0.0038</td>
<td>8.470 ± 6.1230</td>
</tr>
</tbody>
</table>

### C. PID Controller Tuning

After identifying system parameters, we have an accurate simulation model for the Mauch knee / EMG-30 DC motor combination. We proceed by performing the procedure discussed in Section II-A to optimize the PID controller parameters. A chirp input signal with a 10 V amplitude and a 1–10 Hz frequency range is applied to the motor. We optimize the PID controller for various shank lengths.

For each shank length the stochastic optimization algorithms are executed 20 times. The average convergence of each algorithm is illustrated in Fig. 7. SQP shows slow convergence compared to PSO and BBO. PSO shows the fastest convergence, especially during the early iterations.

After 6,000 function evaluations, BBO achieved the best average ISE of 2.307, SQP was second best with 5.112, and PSO was worst with an average ISE of 6.403.

Next we determined how the optimal PID parameters vary with shank length. Figs. 8, 9, and 10 show that PSO, BBO, and SQP give qualitatively similar results, although PSO finds the largest proportional gains. The optimal proportional gain increases with shank length. However, the optimal differential and integral gains have a tendency to decrease with increasing shank length.

It is known that increasing proportional gain results in an increase in the controller output for a given error. On the other hand, shank length is directly proportional to the energy required for the shank to track the reference [28]. The increase in the optimal proportional gain with respect to shank length indicates a requirement to increase the control signal to match the energy demand required to move the larger radial load.

![Average convergence for BBO, SQP, and PSO for tuning the active prosthetic knee PID controller](image)
VI. CONCLUSION AND FUTURE WORK

We investigated the relationship between optimal PID controller parameters and prosthetic leg design. Results show that increasing shank length results in an increase in the optimal proportional gain, and decreases in the optimal integral and differential gains. Evolutionary algorithms and nonlinear quadratic programming techniques were tested and experimentally validated for the identification of the model parameters.

Future work will take the full dynamic model of the prosthetic leg into account. Also, more study is needed to characterize controller characteristics with respect to other features (e.g., foot weight, load distribution on foot, and foot length). Future work will also investigate the optimization of other control techniques, such as LQR and sliding mode.

REFERENCES