Robust Adaptive Impedance Control With Application to a Transfemoral Prosthesis and Test Robot

This paper presents, compares, and tests two robust model reference adaptive impedance controllers for a three degrees-of-freedom (3DOF) powered prosthesis/test robot. We first present a model for a combined system that includes a test robot and a transfemoral prosthetic leg. We design these two controllers, so the error trajectories of the system converge to a boundary layer and the controllers show robustness to ground reaction forces (GRFs) as nonparametric uncertainties and also handle model parameter uncertainties. We prove the stability of the closed-loop systems for both controllers for the prosthesis/test robot in the case of nonscalar boundary layer trajectories using Lyapunov stability theory and Barbalat’s lemma. We design the controllers to imitate the biomechanical properties of able-bodied walking and to provide smooth gait. We finally present simulation results to confirm the efficacy of the controllers for both nominal and off-nominal system model parameters. We achieve good tracking of joint displacements and velocities, and reasonable control and GRF magnitudes for both controllers. We also compare performance of the controllers in terms of tracking, control effort, and parameter estimation for both nominal and off-nominal model parameters. [DOI: 10.1115/1.4040463]

Keywords: robust adaptive impedance control, transfemoral prosthesis, nonscalar boundary layer trajectories

1 Introduction

Prostheses have become progressively important because there are about two million people with limb loss in the U.S. as of 2008 [1]. Amputation could be due to accidents, cancer, diabetes, vascular disease, birth defects, and paralysis. However, the primary cause of lower limb loss is disease—particularly diabetes and other dysvascular etiologies (approximately 75% of all cases) [1,2]. A prosthetic leg can enhance the quality of life and the ability to walk for amputees, so they can regain independence. Amputation could be transfibial (i.e., below knee), transfemoral (i.e., above knee), at the foot, or disarticulation (i.e., through a joint). Prosthetic legs can be generally classified into three different types: passive prostheses do not include any electronic control, active prostheses include motors, and semi-active prostheses are not actively driven by motors [3]. Research efforts over the past few decades have provided advanced prostheses to closely imitate able-bodied gait and to allow greater levels of activity for amputees. Active prostheses provide gait performance that is more similar to able-bodied gait than passive or semi-active prostheses. The first commercially available active transfemoral prosthesis was the Power Knee [3–5]. A combined knee/ankle prosthesis that includes active control at both knee and ankle has been developed by Vanderbilt University but has not yet been commercialized [6]. Much recent research has focused on the control of these prostheses, along with other prostheses [7–12]. Recent research has provided significant developments in modeling and control for prosthetic legs [13–22], and bipedal robots and rehabilitation robots [23–25]. Although direct neural integration and electromyogram signals can be recorded from residual limbs, and the ground reaction force (GRF) can be measured from prosthetic legs to recognize user intent for volitional control of the powered prosthetic legs, in this paper a pair of “classical” feedback control strategies (robust adaptive impedance controllers (RAIC)) are presented to control the robot/prosthesis device using feedback measurements of the joints position and velocity and feedback of the GRF model.

An active prosthesis is essentially a robot that interacts with its human user. The prosthesis can be controlled to behave as an impedance or admittance [26,27]. The consideration of the interaction between a robot and its external environment motivated the development of impedance control [28]. Variable impedance controller is one of the most popular approaches to control powered prosthetic legs, because it can be used in a model independent fashion. However, this control method suffers several shortcomings: tedious impedance parameter tuning, lack of feedback, and passiveness [3,10].

Modeling errors are always present in real-world systems, but robust control approaches can mitigate the effects of modeling errors on system performance and stability [29,30]. Robust controllers achieve performance in spite of model uncertainty, while adaptive controllers achieve performance using learning and adaptation. Nonadaptive controllers generally require prior knowledge of the parameter variation bounds, while adaptive approaches do not.

The advantages of adaptive control, the availability of able-bodied impedance models, and the uncertainty of robot models have motivated the development of impedance model reference adaptive control [31–33]. However, adaptive control methods can cause instability if disturbances, unmodeled dynamics, or unmodeled external forces are too large. Robust control can alleviate instability in such cases [34–39]. Various adaptive and sliding surface approaches have also been used for robotic applications [30,40–44].

The contribution of this paper is two robust model reference adaptive impedance controllers for transfemoral prostheses, the stability analysis of the two controllers, and the investigation of
their performance in simulation. Our control approaches can ensure that the system converges to a reference model in the presence of both parametric and nonparametric uncertainties. In this paper, we present a blending adaptive and nonscalar boundary layer-based robust control to achieve robustness to GRFs (i.e., environmental interactions), system uncertainties, and disturbances, estimation of the unknown parameters, and a stability proof of the proposed methods.

The first controller comprises a RAIC with a tracking-error-based (TEB) adaptation law, which extracts information about the parameters from only the impedance model tracking error. The second controller comprises a robust composite adaptive impedance controller (RCAIC) with bounded-gain forgetting (BGF). Since tracking errors in the joint displacements and prediction error in the joint torques are influenced by parameter uncertainties, RCAIC is designed with TEB/prediction-error-based (PEB) adaptation so that parameter adaptation is driven with both impedance model tracking error and prediction error, which, in turn, provides more accurate estimation of system parameters. More accurate estimation of the system parameters results in a more accurate model, and in turn, RCAIC can achieve better tracking compared with RAIC.

Since our goal is that the two closed-loop systems (one with RAIC and the other with RCAIC) match the biomechanics of able-bodied walking, we use a target impedance model which is based on able-bodied walking. To balance control chatter and performance, we incorporate nonscalar boundary layer trajectories $s_A$ in both controllers. We use these trajectories to turn off the TEB adaptation mechanism to prevent unfavorable parameter drift when the impedance model tracking errors are small and due mostly to noise and disturbances. We define the trajectories $s_A$ so the error trajectories converge to the boundary layers and the controllers show robustness to both parametric and nonparametric uncertainties.

Among adaptive control methods which have already been published, our work most closely resembles Ref. [30] and Ref. [42]. In Ref. [30], a direct adaptive controller is proposed whose adaptation mechanism uses joint tracking errors. The control law in Ref. [30] is a combination of a direct adaptive and robust sliding mode control based on a scalar boundary layer to obtain a tradeoff between control chatter and performance, and to achieve robustness to unmodeled dynamics. Asymptotic stability of the closed-loop system in the case of a scalar boundary layer is shown.

In Ref. [42], a composite adaptive controller is proposed whose adaptation mode uses tracking errors in the joint motion and errors in the predicted filtered torque to derive more accurate system parameters. In addition, a blend of an adaptive feedforward and a proportional–derivative controller is used and exponential stability of the closed-loop system is proven.

Since a robotic system with more than one degree-of-freedom (DOF), including the 3DOF prosthetic controller in this research, can be considered a nonscalar problem with a coupled nature, in this research, we use nonscalar boundary layer trajectories for both control structures.

So, we expand on the work in Ref. [30] by using nonscalar boundary layer trajectories and incorporating impedance control. We prove the asymptotic stability of the system with both controllers, RAIC and RCAIC, using nonscalar boundary layer trajectories, Barbalat’s lemma, and Lyapunov theory. We also extend the work in Ref. [42] by incorporating nonscalar boundary layer trajectories $s_A$ and impedance control so that both augmented robust composite impedance controllers show robustness to nonparametric model uncertainties and environmental interaction forces (which are GRF variations in our case). We then prove the exponential stability of these controllers using nonscalar boundary layer trajectories.

Simulation results illustrate that both proposed systems have good tracking performance, strong robustness to system model parametric and nonparametric uncertainties, and reasonable control signals and GRFs. Furthermore, numerical results show that the RCAIC demonstrates better parameter estimation and tracking in the presence of system parameter variations. When parameter values vary by 30% from nominal values, the RCAIC has 9.5% better reference trajectory tracking and 76% better parameter estimation, but 9.9% greater control magnitude than RAIC.

The paper is organized as follows: Sec. 2 describes the model of the transfemoral prosthesis and the robotic test system. Section 3 presents the controller structures and proves their stability. Section 4 presents simulation results. Section 5 presents discussion, concluding remarks, and future work.

## 2 Prosthetic Leg Model

### 2.1 Test Robot/Transfemoral Prosthetic Leg

Our system model includes a test robot and a transfemoral prosthesis. The system includes three links and three degrees-of-freedom. This prismatic-revolute-revolute model is shown in Fig. 1. Human hip motion and thigh motion is emulated by the robot. The knee and shank represent the prosthesis. The vertical motion emulates human (or test robot) vertical hip motion, the first axis emulates human (or test robot) thigh motion, and the second axis is angular knee (prosthesis) motion.

Note that the thigh and knee angles are strictly 1DOF in the sagittal walking plane. The transverse DOFs, including addiction and abduction, are also important, but are of secondary consideration; the first-order of importance is motion in the direction of walking, and so that is what this paper focuses on. The construction of the test robot is detailed in Ref. [45]. Figure 1 also shows the prototype prosthesis that we developed at Cleveland State University, which is detailed in Refs. [46] and [47]. This is a generic lower limb prosthesis that has a chord mechanism at the knee and a supercapacitor-based energy regeneration.

The three degrees-of-freedom system model can be written as follows:

$$Mq\dot{q} + C(q, \dot{q})q + g + R = u - T_e$$  \hspace{1cm} (1)

where $q^T = [q_1, q_2, q_3]$ comprises the generalized displacements ($q_1$ is vertical displacement, $q_2$ is thigh angle, and $q_3$ is prosthesis knee angle); $M(q)$ is the inertia matrix; $C(q, \dot{q})$ is the Coriolis and Centripetal matrix; $g(q, \dot{q})$ is the vector gravity; $R(q, \dot{q})$ is the nonlinear damping vector; $T_e = J^T F$ is the effect of the combined horizontal ($F_x$) and vertical ($F_z$) components of the GRF on each joint, where $J$ is the Jacobian matrix and $F = [F_x, F_z]^T$ is the GRF vector; $u$ comprises the active control force at the hip and the active control torques at the thigh and the prosthetic knee. It should be noted that “thigh torque” is also known as “hip torque” in the biomechanical literature.

We assume that the positions and velocities of each joint are measured accurately with encoders and differentiators. More details about the sensors are provided in Ref. [45]. However, there is no need to measure joint accelerations for our proposed controllers.

Although in a real-world prosthesis application, the hip force and thigh torque are controlled by the human amputee, in this research, they are control inputs of the test robot shown in Fig. 1, which emulates human hip and thigh motions. Also, real-time measurements of $q_1$ and $q_3$ are challenging in amputees, but in this paper, they can easily be measured by incremental encoders.

### 2.2 Ground Reaction Forces Model

The prosthetic test robot walks on a treadmill, which we model as a mechanical stiffness [16]. We model the vertical component of the GRF ($F_z$) for the foot-treadmill contact as

$$F_z = \begin{cases} 0, & L_1 < s_z \\ -k_b(s_z - L_2), & L_2 > s_z \end{cases}$$  \hspace{1cm} (2)

where $k_b$ is the belt stiffness; $s_z$ is the treadmill standoff (i.e., the vertical distance from the origin of the world frame ($x_0, y_0, z_0$) to
the belt); $L_z$ is the vertical position of bottom of the foot in the world frame, which is given as follows (see Fig. 1):

$$L_z = q_1 + l_2 \sin(q_2) + l_3 \sin(q_2 + q_3)$$

where $l_2$ and $l_3$ are the length of the thigh and shank, respectively. There is much less slack in the $x$-direction than the $z$-direction, so we consider slack only in $z$-direction [46]. The horizontal component of the GRF ($F_x$) can be modeled by an approximation of Coulomb friction as [48]

$$F_x = -\beta v_x \left( \frac{1 - e^{-v_x/\nu_x}}{1 + e^{-v_x/\nu_x}} \right)$$

where $\beta$ is the belt friction coefficient; $v_x$ is scaling factor; $v_x$ is the velocity of the foot-treadmill contact relative to the treadmill, such that

$$v_x = -\dot{q}_2 (l_2 \sin(q_2) + l_3 \sin(q_2 + q_3)) - \dot{q}_3 (l_3 \sin(q_2 + q_3)) - v_t$$

where $v_t$ is the treadmill speed. Based on Eq. (2), we divide one stride into two phases: swing phase, where $L_z < s_z$, and stance phase, where $L_z > s_z$. Therefore, we have zero $F_z$ and zero GRF in the swing phase, and when the point foot hits the ground (stance phase), GRF appears as the belt stiffness times the belt deflection, $T_e = J^T F$ is due to the effect of GRF on each joint and is given as follows [16,17]:

$$T_e = \begin{bmatrix} F_z \\
F_z (l_2 \cos(q_2) + l_3 \cos(q_2 + q_3)) - F_x (l_2 \sin(q_2) + l_3 \sin(q_2 + q_3)) \\
F_z (l_3 \cos(q_2 + q_3)) - F_x (l_3 \sin(q_2 + q_3)) \\
0 -l_2 \sin(q_2) - l_3 \sin(q_2 + q_3) - l_3 \sin(q_2 + q_3) \\
1 l_2 \cos(q_2) + l_3 \cos(q_2 + q_3) l_3 \cos(q_2 + q_3) \\
\end{bmatrix}$$

$$J = \begin{bmatrix} f_1 & \tau_{\text{hip}} & \tau_{\text{knee}} \end{bmatrix}$$

Robot dynamics can be linearly parameterized by a model regressor $Y'(q, \dot{q}, \ddot{q}) \in R^{n \times r}$ and parameter vector $p \in R^r$, so the right side of Eq. (1) can be written in the following form:

$$M \ddot{q} + C \dot{q} + g + R = Y'(q, \dot{q}, \ddot{q}) p$$

where $Y'(q, \dot{q}, \ddot{q})$ is a function of joint displacements, velocities, and accelerations; $n$ is the number of links ($n = 3$ in this paper; see Fig. 1); $r$ is the number of parameter vector elements (here $8$ in this paper as shown below). The regressor $Y'(q, \dot{q}, \ddot{q})$ and the parameter $p$ have many realizations; one such possibility is

$$Y'(q, \dot{q}, \ddot{q}) = \begin{bmatrix} \dot{q}_1 - g Y'_{12} & Y'_{13} & 0 & 0 & 0 & 0 & \text{sgn}(\ddot{q}_1) \\ 0 & 0 & Y'_{23} & \ddot{q}_2 & Y'_{25} & \ddot{q}_3 & 0 \\ 0 & 0 & 0 & Y'_{33} & \ddot{q}_2 + \ddot{q}_3 & 0 & 0 \end{bmatrix}$$

$$Y'_{12} = \ddot{q}_2 \cos(q_2) - \ddot{q}_2 \sin(q_2)$$

$$Y'_{13} = (\ddot{q}_2 + \ddot{q}_3) \cos(q_3 + q_2) - (2 \ddot{q}_2 \ddot{q}_3 + \ddot{q}_3^2) \sin(q_3 + q_2)$$

$$Y'_{22} = (\ddot{q}_1 - g) \cos(q_2)$$

$$Y'_{23} = Y'_{33} = (\ddot{q}_1 - g) \cos(q_3 + q_2)$$

$$Y'_{25} = (2 \ddot{q}_2 + \ddot{q}_3) \cos(q_3) - (2 \ddot{q}_2 \ddot{q}_3 + \ddot{q}_3^2) \sin(q_3)$$

$$Y'_{35} = \ddot{q}_2 \cos(q_3) + \sin(q_3) \ddot{q}_3^2$$

$$p = \begin{bmatrix} m_1 + m_2 + m_3 & m_3 l_2 + m_2 l_3 + m_2 c_2 \\ m_3 c_3 & m_1 c_3 \\ l_2 c + l_3 c + m_2 c_2^2 + m_3 c_2^2 + m_3 l_2^2 + m_3 l_3^2 + 2 m_2 c_2 l_2 \\ m_3 c_2 l_2 \\ m_3 c_3^2 + l_3 c \\ b & f \end{bmatrix}$$

3 Robust Adaptive Impedance Control

We design two separate nonlinear robust adaptive impedance controllers using nonscalar boundary layers and sliding surfaces to track hip displacement and knee and thigh angles in spite of parametric and nonparametric uncertainties. Both controllers use the same
control laws, same target impedance models, and same nonscalar boundary layer trajectories but different adaptation laws. In the first controller, we design a RAIC with a TEB adaptation law, which extracts information about the parameters from the impedance model tracking error. In the second controller, we propose a RCAIC with BOF. Since impedance model tracking errors in the joint displacements and prediction error in the joint torques are influenced by parameter uncertainties, in RCAIC we design a TEB/PEB adaptation law which drives parameter adaptation using both impedance model tracking error and prediction error to achieve more accurate estimation of the system parameters.

3.1 Target Impedance Model. The robot/prosthesis system interacts with the environmental admittance, so if we want to have a system that is well-matched with the mechanical characteristics of the environment, the closed-loop system should behave as an impedance. In this way, we can achieve a tradeoff between performance and GRF.

We desire the closed-loop systems with both RAIC and RCAIC to emulate the biomechanics of able-bodied walking [16,49]. We thus define a target impedance model [32] with characteristics similar to able-bodied walking.

\[ M_r(q_r - q_\text{d}) + B_r(q_r - q_\text{d}) + K_r(q_r - q_\text{d}) = -T_e \]  

(11)

The reference mass \( M_r \), damping coefficient \( B_r \), and spring stiffness \( K_r \) are positive definite matrices, while \( q_\text{d} \in \mathbb{R}^n \) is the state of the reference model and \( q_\text{d} \in \mathbb{R}^n \) is the reference trajectory. We assume that the matrices are diagonal

\[
egin{align*}
M_r &\in \mathbb{R}^{n \times n} = \text{diag}(M_{11}, M_{22}, \ldots, M_{nn}) \\
B_r &\in \mathbb{R}^{n \times n} = \text{diag}(B_{11}, B_{22}, \ldots, B_{nn}) \\
K_r &\in \mathbb{R}^{n \times n} = \text{diag}(K_{11}, K_{22}, \ldots, K_{nn})
\end{align*}
\]

(12)

3.2 Control Law. In Eq. (9), the regressor depends on acceleration. However, acceleration measurements are typically noisy, so it might not be convenient to use \( Y(q, \dot{q}, \ddot{q}) \) in real time. To avoid the use of acceleration, we define error vector \( s \) and signal vector \( v \) [40,41,50]

\[
s = \dot{e} + \lambda e \\
v = \ddot{q} - \lambda e \\
e = q - q_\text{r}
\]

(13)-(15)

where \( \lambda \) is a positive scalar tuned by the user. The joint acceleration measurements can be very noisy, so an acceleration-free controller regressor using signal vector \( v \) in Eq. (14) in place of the model regressor in Eq. (9)

\[
M\ddot{q} + C\dot{q} + g + R = Y(q, \dot{q}, v, \ddot{v})p
\]

(16)

where \( Y(q, \dot{q}, v, \ddot{v}) \) is a linear function, one realization of which is given as

\[
Y(q, \dot{q}, v, \ddot{v}) = \begin{bmatrix}
\dot{v}_1 - g & Y_{12} & Y_{13} & 0 & 0 & 0 & 0 & 0 & \text{sgn}(\dot{q}_1) \\
0 & Y_{22} & Y_{23} & \dot{v}_2 & Y_{25} & \dot{v}_3 & \dot{q}_2 & 0 & 0 \\
0 & 0 & Y_{33} & 0 & Y_{35} & \dot{v}_3 & \dot{v}_2 & \dot{v}_3 & 0 & 0
\end{bmatrix}
\]

By substituting Eqs. (13)-(15) in Eq. (1), we rewrite the model as

\[
M\ddot{s} + Cs + g + R + M\ddot{v} + Cv = u - T_e
\]

(18)

Fig. 1 The left figure shows the test robot/transfemoral prosthetic leg with a passive ankle; the right figure shows the 3DOF unified model with a prosthesis foot. Human hip and thigh motions are emulated by a prosthesis test robot where the calf represents the prosthesis device with rigid ankle and foot. A treadmill belt serves as the walking surface. When the foot is in contact with the treadmill belt, the GRF is nonzero.
Since Eq. (1) is a second-order system, the error vector of Eq. (15) can be obtained from the first-order sliding surface

$$s = \left( \frac{d}{dt} + \lambda \right) e$$

(19)

where $s$ includes $n$ elements. Perfect impedance model tracking $q = q_e$ ($e = 0$) implies that $s = 0$. To reach the sliding manifold $s = 0$, the following reaching condition must be satisfied [30]:

$$\text{sgn}(s)\dot{s} \leq -\gamma$$

(20)

This vector inequality is taken one element at a time, and $\gamma$ is an $n$-element vector denoted as $\gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n]^T$ where $\gamma_i > 0$ is a design parameter. Eq. (20) shows that in the worst case, $\text{sgn}(s)\dot{s} = -\gamma$, so we calculate the worst-case reaching time of the tracking error trajectory as

$$\int_0^\theta \text{sgn}(s)ds = -\gamma \int_0^\theta dt \rightarrow |s(0)|\text{sgn}(s) = \gamma T \rightarrow T = s(0)/\gamma$$

(21)

This equation gives $n$ different reaching times, $s(0)$ is the error at the initial time, and the quotient $s(0)/\gamma$ is defined one element at a time. We can see from Eq. (21) that a larger $\gamma$ gives smaller reaching times $T$. The system parameters are not known, so we use a controller [30] to handle parameter uncertainty and to satisfy the condition of Eq. (20):

$$u = \hat{M}\dot{v} + \hat{C}v + \hat{g} + \hat{R} + \hat{T}_e - K_d\text{sgn}(s)$$

(22)

where $\hat{M}, \hat{C}, \hat{g}, \hat{R},$ and $\hat{T}_e$ are estimates of $M, C, g, R,$ and $T_e$, and $K_d$ is a tuning matrix denoted as $K_d = \text{diag}(K_{d1}, K_{d2}, \ldots, K_{dn})$, where $K_{di} > 0$. Note that $\text{sgn}(s)$ is discontinuous, which means that it would result in control chattering; therefore, we replace it with the saturation function sat($s$/diag($\phi$)) (see Fig. 2). The division and saturation operations in sat($s$/diag($\phi$)) are taken one element at a time. The term diag($\phi$) is an $n$-element vector. This all results in a modification of the controller of Eq. (22):

$$u = \hat{M}\dot{v} + \hat{C}v + \hat{g} + \hat{R} + \hat{T}_e - K_d\text{sat}(s-/diag(\phi))$$

(23)

The diagonal elements of $\phi$ are the widths of the saturation function. The control law of Eq. (23) includes two parts. The first part, $\hat{M}\dot{v} + \hat{C}v + \hat{g} + \hat{R}$, is an adaptive term that handles uncertain parameters. The second part, $\hat{T}_e - K_d\text{sat}(s-/diag(\phi))$, is a robustness term that satisfies Eq. (20) and the variations of the external inputs $T_e$ as nonparametric uncertainties. We substitute Eq. (23) into Eq. (18) and define $\hat{M} = \hat{M} - M$, $\hat{C} = \hat{C} - C$, $\hat{g} = \hat{g} - g$, $\hat{R} = \hat{R} - R$, and $\hat{p} = \hat{p} - p$, to derive the closed-loop system

$$\dot{s} + Cs + K_d\text{sat}(s-/diag(\phi)) + (T_e - \hat{T}_e) = (\hat{M}\dot{v} + \hat{C}v + \hat{g} + \hat{R})$$

(24)

where $\hat{p}$ is the estimate of $p$.

We separate the right side of Eq. (24) into two parts: the regressor $Y(q, \dot{q}, v, \dot{v})$ and the parameter estimation error $\hat{p}$. We can, thus, write Eq. (24) in the following regressor (linear parametric) form:

$$\dot{s} + Cs + K_d\text{sat}(s-/diag(\phi)) + (T_e - \hat{T}_e) = Y(q, \dot{q}, v, \dot{v})\hat{p}$$

(25)

### 3.3 Nonscalar Boundary Layer Trajectories

One of the challenges with adaptive control is that in the presence of nonparametric uncertainties such as noise and disturbances, and also in the presence of large adaptation gains and reference trajectories, the estimated parameters are prone to oscillate and grow without bound because of instability in the control system. This phenomenon is known as parameter drift. However, if the model regressor $Y(q, \dot{q}, \dot{v})$ satisfies persistent excitation (PE) conditions, the adaptive control scheme exhibits robustness against nonparametric uncertainties and unmodeled dynamics, and parameter drift can be avoided [30,50].

To turn off the TEB adaptation mechanism to prevent unfavorable parameter drift when the impedance model tracking errors are small and due mostly to noise and disturbances, we incorporate nonscalar boundary layer trajectories $s_a$ into both controllers RAIC and RCAIC. We define these trajectories to balance control chatter and performance. Furthermore, we define the trajectories $s_a$ so the error trajectories converge to the boundary layers and both proposed controllers show robustness to nonparametric uncertainties. We define these boundary trajectories $s_a$ as follows [30]:

$$s_a = \begin{cases} 0, & |s| \leq \text{diag}(\phi) \\ s - \text{diag}(s-/diag(\phi)), & |s| > \text{diag}(\phi) \end{cases}$$

(26)

Note that $s_a$ is an $n$-element vector. We call the region $|s| \leq \text{diag}(\phi)$ the boundary layer, where the inequality is taken one element at a time. Note that the diagonal elements of $\phi$ comprise the thickness values of the boundary layer and are denoted as $\phi = \text{diag}(\phi_1, \phi_2, \ldots, \phi_n)$, where tunable $\phi_i > 0$. We illustrate $s_a$ and sat($s-/diag(\phi)$) for a single dimension in Fig. 2.

### 3.4 Robust Adaptive Impedance Controller. RAIC

The RAIC uses the control law in Eq. (23), nonscalar boundary layer trajectories in Eq. (26), and the TEB adaptation law, so the prosthesis/RAIC combination converges to the target impedance model in Eq. (11). The TEB adaptation law can be presented as

$$\dot{\hat{p}} = -\mu^{-1}Y^T(q, \dot{q}, v, \dot{v})s_a$$

(27)

where $\mu \in R^{n\times r}$ is a positive definite matrix with diagonal elements, which is adjusted by the user.

**Theorem 1.** Consider the following scalar positive definite Lyapunov function [50]

$$V(s_a, \hat{p}) = \frac{1}{2} (s_a^T Ms_a) + \frac{1}{2} (\hat{p}^T \hat{p})$$

(28)

where $\mu$ is a design parameter such that $\mu = \text{diag}(\mu_1, \mu_2, \ldots, \mu_r)$, with $\mu_i > 0$. The closed-loop system using RAIC results in $V(s_a, \hat{p}) \rightarrow 0$ as $t \rightarrow \infty$. That is, the closed-loop systems are asymptotically stable. The error vector $s$ converges to the boundary layer, which implies convergence of the closed-loop system to the target impedance model.

**Proof of Theorem 1:** See Appendix A.

### 3.5 Robust Composite Adaptive Impedance Controller.

The RCAIC uses the same control law in Eq. (23) and nonscalar boundary layer trajectories in Eq. (26) as the RAIC uses but uses a

![Fig. 2 Saturation function and s_a in one dimension](https://example.com/saturation.png)
different adaptation law, i.e., the TEB/PEB mechanism, so the prosthesis/RCAIC combination converges to the target impedance model in Eq. (11). In the TEB adaptive controller (RAIC), the adaptation law extracts information about the parameters only from the impedance model tracking error. However, the tracking error is not the only source of parameter information; prediction error also contains parameter information. Therefore, by using a combination of the impedance model tracking and prediction errors, the performance of the adaptive controller can be improved. For the RCAIC, a TEB/PEB adaptation law is introduced as follows [30,50]:

\[
\hat{p} = -P(t)Y^T(q, \dot{q}, v, \dot{v})s_{\lambda} + W^T R\hat{p}
\]  

(29)

where \( R = d_{\text{un}} \) is a positive definite diagonal weighting matrix that indicates how much the adaptation law uses the prediction error (\( d \) is a positive constant); \( P(t) \) is time-varying adaptation gain; \( W \) is a filtered version of the model regressor matrix \( Y(q, \dot{q}, \ddot{q}) \) given in Eq. (9), where this filtering is introduced to avoid the need for joint acceleration in the regressor [50]; and \( e_p \) is the prediction error and is calculated from \( W(q, \dot{q})\hat{p} \) (details will be presented later in this section). The filtering can be done with a first-order stable filter as follows:

\[
W(q, \dot{q}) = \frac{c}{s + c} Y(q, \dot{q}, \ddot{q})
\]  

(30)

where \( c > 0 \). To filter in the time domain, we convolve both sides of Eq. (1) with the impulse response of \( \frac{c}{s + c} \) (that is, \( w(t) = ce^{-ct} \)):

\[
\int_0^t w(t-h)M\dot{q} + C\ddot{q} + g + R\ddot{h} dh = \int_0^t w(t-h)[u - T_e] dh
\]  

(31)

The first part of Eq. (31), \( \int_0^t w(t-h)M\dot{q}dh \), can be written as follows:

\[
\int_0^t w(t-h)M\dot{q}dh = w(t)M\dot{q} + \int_0^t \frac{d}{dh}[w(t-h)]M\dot{q} dh - \int_0^t \frac{d}{dh} w(t-h)M\dot{q} dh
\]  

(32)

That is, convolving the left-hand side of Eq. (31) can be interpreted as filtering that side and is equal to \( W(q, \dot{q})p \), so that

\[
y(t) = W(q, \dot{q})p = w(0)M\dot{q} - w(t)M\dot{q}(0)\dot{q}(0) - \int_0^t [w(t-h)M\dot{q} + \frac{d}{dh}[w(t-h)]M\dot{q} dh + \int_0^t w(t-h)[C\ddot{q} + g + R]\ddot{h} dh
\]  

(33)

where \( y(t) \) is the filtered version of the right side of Eq. (1) and is given as follows:

\[
y(t) = \int_0^t w(t-h)[u - T_e] dh
\]  

(34)

The estimated value of \( y(t) \) can be written as follows:

\[
\hat{y}(t) = W(q, \dot{q})\hat{p}
\]  

(35)

Therefore, the prediction error \( e_p \) is derived as

\[
e_p = \hat{y}(t) - y(t) = W(q, \dot{q})\hat{p}
\]  

(36)

It is important to note that past data are generated from past parameter values, and the algorithm should therefore pay less attention to past data when generating current parameter estimates. Therefore, exponential data forgetting is advisable for estimating time-varying parameters. The composite adaptation law in Eq. (29) can benefit from an exponentially forgetting least-squares gain update for \( P(t) \) as follows [42,50]:

\[
\frac{d}{dt}(P^{-1}) = -\vartheta(t)P^{-1} + W^T(t)W(t)
\]  

(37)

where \( \frac{d}{dt} \vartheta(t) \geq 0 \) denotes the time-varying forgetting factor. To benefit from data forgetting and to avoid unboundedness in \( P(t) \), the BGF method can be used to tune the time-varying forgetting factor \( \vartheta(t) \) as follows [50]:

\[
\vartheta(t) = \vartheta_0 \left( 1 - \frac{\|P\|}{K_0} \right)
\]  

(38)

where \( \vartheta_0 \) is the maximum forgetting rate; \( K_0 \) is the upper bound of \( P(t) \); and \( P(0) \) must be smaller than \( K_0 I \). The second part of the TEB/RCAIC adaptation law in Eq. (29) can be written as

\[
\hat{p} = -P(t)W^TR\hat{p}
\]  

(39)

Solving Eq. (39) gives

\[
\hat{p}(t) = \hat{p}(0) \exp \left( \int_0^t -P(t)W^TRW(t) dt \right)
\]  

(40)

Therefore, \( \hat{p} = \hat{p} - p \) will exponentially converge to zero if \( W(q, \dot{q}) \) is PE. The speed of convergence can be heavily dependent on the magnitude of the adaptation gain. \( W(q, \dot{q}) \) must satisfy the following PE condition:

\[
\lim_{t \to \infty} \int_0^t -W^T(t)W(t) dt = \infty
\]  

(41)

Therefore, \( \hat{p} \) will exponentially converge to zero for nonzero and constant \( W \). It is interesting to note that when \( W \) is not PE, \( \hat{p} \) cannot converge to zero, even if there are no nonparametric uncertainties, and the robustness property cannot be guaranteed. In this procedure, the time-varying forgetting factor is tuned so that data forgetting is active when \( W(t) \) is PE and it is off when \( W(t) \) is not PE. From Eq. (38), \( \|P\| \) shows the level of PE of \( W(t) \) so that if \( \|P\| \) decreases, \( W(t) \) is strongly PE (\( \vartheta(t) = \vartheta_0 \)), and if \( \|P\| \) increases, \( W(t) \) is weakly PE. In the BGF composite controller, \( \hat{p} \) and \( P(t) \) are upper bounded, and if \( W(t) \) is strongly PE, then \( \hat{p} \) exponentially converges to zero, \( P(t) \) is upper and lower bounded by positive numbers, and \( \vartheta(t) > \vartheta_0 > 0 \).

**Theorem 2.** Consider the scalar positive definite Lyapunov function

\[
V(s, \hat{p}) = \frac{1}{2} (s_{\lambda}^T M s_{\lambda}) + \frac{1}{2} \hat{p}^T P^{-1} \hat{p}
\]  

(42)

The controller of Eq. (23), when used in conjunction with the update law of Eq. (29) and applied to the system of Eq. (1), results in \( V(s, \hat{p}) \to 0 \) as \( t \to \infty \), which means the prosthesis/RCAIC combination is globally exponentially stable. The error vector \( s \) converges to the boundary layer, indicating perfect estimation of the system parameters and convergence of the closed-loop system to the target impedance model.

**Proof of Theorem 2:** See Appendix B.

3.6 System Convergence. To get a feeling for the RAIC/RCAIC structure, consider the general structure of Fig. 3. To
show that both proposed controller structures RAIC and RCAIC result in closed-loop systems that converge to the target impedance model, we use Eqs. (13)–(15) to write the closed-loop system as

\[ \ddot{q} + (\mathbf{C} + \mathbf{M}\dot{\mathbf{q}}) + \mathbf{K}_{r} \mathbf{q} = \mathbf{M}\dot{\mathbf{q}} + \mathbf{C}\mathbf{q} - \ddot{\mathbf{R}} + \mathbf{M}\dot{\mathbf{s}} + \mathbf{C}\mathbf{e} + (\mathbf{T}_{e} - \mathbf{T}_{e}) \]  

(43)

From Eq. (11) we have the target impedance model

\[ M_{r}\ddot{q}_{r} + B_{r}\dot{q}_{r} + K_{r}q_{r} = M_{r}\dot{q}_{r} + B_{r}\dot{q}_{d} + K_{r}q_{d} - T_{r} \]  

(44)

From Theorems 1 and 2, \( s_{3} \) → 0 as \( t \) → ∞ so the trajectories of \( s \) are bounded in the boundary layers. Now since \( s \) is bounded, \( e \) and \( \dot{e} \) are bounded. The boundedness of \( q_{r}, \dot{q}_{r}, e, \) and \( \dot{e} \) implies that \( q \) and \( \dot{q} \) are bounded, which in turn implies that the right side of Eq. (43) is bounded, just as the right side of Eq. (44) is bounded.

It is seen that the closed-loop system in Eq. (43) has the same structure as the impedance model of Eq. (44), which means both proposed controllers result in closed-loop systems that converge to the target impedance model of Eq. (44); where comparing Eq. (43) with Eq. (44) gives \( M_{r} = \mathbf{M}, B_{r} = \mathbf{C}, \) and \( K_{r} = \mathbf{K} \).

We see that the proposed control law in Eq. (23) for both RAIC and RCAIC drives the closed-loop system in Eq. (25) to match the impedance model in Eq. (11).

### 4 Simulation Results

#### 4.1 Experimental Reference Trajectory

The reference trajectory is obtained from the motion studies lab (MSL) of the Cleveland Veterans Affairs Medical Center (VAMC) [11]. In order to calculate three-dimensional joint angles, a three-dimensional model was constructed from 47 reflective markers placed on the research participants. The research participants were volunteers in this study, which was approved by the Institutional Review Board of the Cleveland VAMC. Data were collected at a specific walking speed. The research participants walked on a treadmill for 10–30 s trials while kinematic and kinetic data were collected at their preferred walking speed. This speed was determined using previous methods [51], which allowed for acclimating to the treadmill. Moreover, all of the research participants had previous treadmill experience. The kinematic data were collected at 100 Hz via a 16 camera passive marker motion capture system (Vicon, Oxford Metrics, UK) with the markers mounted according to the Human Body Model (Motek, Amsterdam, The Netherlands). In addition, GRFs were collected at 1000 Hz via two force plates within the treadmill (ADAL3DM-F-COP-Mz, Tecmachine, France). For data processing, 100 frames were taken from a standing trial for initialization of the subject-specific model comprised 18 body segments and 46 kinematic degrees-of-freedom. The kinematics and the ground reaction forces were the input for an inverse dynamic analysis after low pass filtering at 6 Hz with a

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_{2} )</td>
<td>Treadmill standoff (Eq. (2))</td>
<td>0.905</td>
<td>m</td>
</tr>
<tr>
<td>( k_{p} )</td>
<td>Belt stiffness (Eq. (2))</td>
<td>37000</td>
<td>N/m</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Belt friction coefficient (Eq. (4))</td>
<td>0.2</td>
<td>—</td>
</tr>
<tr>
<td>( v_{c} )</td>
<td>Scaling factor (Eq. (4))</td>
<td>0.05</td>
<td>m/s</td>
</tr>
<tr>
<td>( v_{t} )</td>
<td>Treadmill speed (Eq. (4))</td>
<td>1.25</td>
<td>m/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Nominal value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{1} )</td>
<td>Mass of link 1</td>
<td>40.5969</td>
<td>kg</td>
</tr>
<tr>
<td>( m_{2} )</td>
<td>Mass of link 2</td>
<td>8.5731</td>
<td>kg</td>
</tr>
<tr>
<td>( m_{3} )</td>
<td>Mass of link 3</td>
<td>2.29</td>
<td>kg</td>
</tr>
<tr>
<td>( l_{2} )</td>
<td>Thigh length</td>
<td>0.425</td>
<td>m</td>
</tr>
<tr>
<td>( l_{3} )</td>
<td>Length of joint to bottom of shoe</td>
<td>0.527</td>
<td>m</td>
</tr>
<tr>
<td>( c_{2} )</td>
<td>Center of mass on thigh</td>
<td>0.09</td>
<td>m</td>
</tr>
<tr>
<td>( c_{3} )</td>
<td>Center of mass on shank</td>
<td>0.32</td>
<td>m</td>
</tr>
<tr>
<td>( f )</td>
<td>Sliding friction in link 1</td>
<td>83.33</td>
<td>N</td>
</tr>
<tr>
<td>( b )</td>
<td>Rotary actuator damping</td>
<td>9.75</td>
<td>N m/s</td>
</tr>
<tr>
<td>( I_{2} )</td>
<td>Rotary inertia of link 2</td>
<td>0.138</td>
<td>kg m²</td>
</tr>
<tr>
<td>( I_{3} )</td>
<td>Rotary inertia of link 3</td>
<td>0.0618</td>
<td>kg m²</td>
</tr>
<tr>
<td>( g )</td>
<td>Acceleration of gravity</td>
<td>9.81</td>
<td>m/s²</td>
</tr>
</tbody>
</table>
second-order low pass Butterworth filter. The data were finally processed to solve the skeletal motion and to compute the inverse dynamics for each model.

### 4.2 Prosthesis Test Robot Model, Controllers, and Target Impedance Model Parameters

Here we demonstrate the performance of RAIC and RCAIC with simulation. In the prosthesis test model considered here, we have \( q \in \mathbb{R}^3 \), so target impedance model coefficients presented in Eq. (12) can be written as \( M_i = \text{diag}(M_{11} \ M_{22} \ M_{33}) \), \( B_i = \text{diag}(B_{11} \ B_{22} \ B_{33}) \), and \( K_i = \text{diag}(K_{11} \ K_{22} \ K_{33}) \). To obtain critically-damped responses (two equal roots for each joint displacement) in the reference impedance model of Eq. (11), we set \( B_i = 2\sqrt{K_i/M_i} \) and the two roots are both equal to \(-\sqrt{K_i/M_i} \ (i = 1, 2, 3)\). To obtain two different real roots, \( B_i = 2\sqrt{K_i/M_i} \). Here, we use a reference impedance model with roots \(-11 \) and \(-88 \) for the thigh, \(-5 \) and \(-94 \) for the knee, and \(-3 \) and \(-497 \) for the hip. These values provide a reference impedance model that is stable, performs similar to able-bodied walking, provides a good reference model tracking, and provides control signals and GRFs with the same order of magnitude as able-bodied ones. We, thus, obtain the following reference impedance matrices:

\[
M_i = \text{diag}(10, 10, 10) \\
K_i = \text{diag}(15000, 10000, 5000) \\
B_i = \text{diag}(5000, 1000, 1000)
\]

We assume that the treadmill parameters (i.e., GRFs parameters) are constant and listed in Table 1. We suppose that the prosthesis test robot parameters are partly unknown and can vary by up to 30% from their nominal values [16]. The nominal system parameters are shown in Table 2. The initial state is \( 0.019 \ 1.13 \ 0.09 \ 0.09 \ 0 \ 1.6 \). After some experimentation, we achieve good performance for RAIC and RCAIC with the design parameters in Table 3. As seen from Table 3, the controller design parameters are round numbers, which means they are defined as follows:

\[
\begin{align*}
\phi & = \text{Boundary layer thicknesses (Eq. (26))} \\
K_r & = \text{Robust term coefficients (Eq. (22))} \\
\mu & = \text{Adaptation rate (Eq. (27))} \\
\lambda & = \text{Sliding term coefficients (Eq. (13))} \\
\rho & = \text{Boundary layer thicknesses (Eq. (26))} \\
\vartheta & = \text{Robust term coefficients (Eq. (22))} \\
\alpha & = \text{Sliding term coefficients (Eq. (13))} \\
\theta & = \text{Maximum forgering rate (Eq. (38))}
\end{align*}
\]

\[
P(0) = \text{Upper bound of the adaptation gain (Eq. (38))}
\]

The controllers design parameters are as follows:

<table>
<thead>
<tr>
<th>Controller type</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAIC</td>
<td>( \phi )</td>
<td>Boundary layer thicknesses (Eq. (26))</td>
<td>0.5</td>
</tr>
<tr>
<td>RAIC</td>
<td>( K_r )</td>
<td>Robust term coefficients (Eq. (22))</td>
<td>100</td>
</tr>
<tr>
<td>RAIC</td>
<td>( \mu )</td>
<td>Adaptation rate (Eq. (27))</td>
<td>0.011</td>
</tr>
<tr>
<td>RAIC</td>
<td>( \lambda )</td>
<td>Sliding term coefficients (Eq. (13))</td>
<td>100</td>
</tr>
<tr>
<td>RAIC</td>
<td>( \rho )</td>
<td>Boundary layer thicknesses (Eq. (26))</td>
<td>0.5</td>
</tr>
<tr>
<td>RAIC</td>
<td>( \vartheta )</td>
<td>Robust term coefficients (Eq. (22))</td>
<td>100</td>
</tr>
<tr>
<td>RAIC</td>
<td>( \alpha )</td>
<td>Sliding term coefficients (Eq. (13))</td>
<td>100</td>
</tr>
<tr>
<td>RCAIC</td>
<td>( \theta )</td>
<td>Maximum forgering rate (Eq. (38))</td>
<td>5</td>
</tr>
<tr>
<td>RCAIC</td>
<td>( P(0) )</td>
<td>Upper bound of the adaptation gain (Eq. (38))</td>
<td>400</td>
</tr>
<tr>
<td>RCAIC</td>
<td>( c )</td>
<td>Initial condition of the adaptation gain (Eq. (37))</td>
<td>1</td>
</tr>
<tr>
<td>RCAIC</td>
<td>( D )</td>
<td>Weighting constant (Eq. (29))</td>
<td>2</td>
</tr>
</tbody>
</table>

The total cost is a combination of the total desired trajectory tracking cost in Eq. (49), and the total control cost in Eq. (50):
We define a cost function to evaluate the estimation of the parameter vector \( p \in \mathbb{R}^8 \) presented in Eq. (10) as

\[
\text{Cost} = \text{Cost}_E + \text{Cost}_U
\] (51)

\[
\text{RMSP}_k = \sqrt{\frac{1}{T} \int_0^T \left( \hat{p}_k - p_{0k} \right)^2 dt}, \quad k = 1, \ldots, 8
\] (52)

where \( \hat{p}_k \) and \( p_{0k} \) are \( k \)th elements of the estimated parameter vector and the true parameter vector, respectively. The normalized and total estimation costs are given as follows:

\[
\text{Cost}_{pk} = \frac{\text{RMSP}_k}{\max_{i \in [1, T]} |p_{0i}|}
\] (53)

\[
\text{Cost}_P = \sum_{k=1}^{8} \text{Cost}_{pk}
\] (54)

4.4 Simulation Results

4.4.1 Tracking Performance. Figure 4 compares the states of the system with RAIC and RCAIC and the reference trajectories \((q_D)\) when all system parameters are equally varied by 30\% from their nominal values (Table 2).

It should be noted that to consider the tracking sensitivity to the system parameters, a sensitivity analysis is required in which the parameters should be perturbed one by one not altogether. Figure 4 shows that both controllers demonstrate robustness and also walking behavior of the prosthesis is similar to human-like walking. Although, Theorems 1 and 2 guarantee that the states of the system accurately track the model reference trajectory \((q_r)\), Fig. 4 compares the states of the system with the reference trajectories.
The reason is to show the flexibility of the prosthesis walking in following the desired trajectories provided by the impedance model of Eq. (11). In this paper, the actual hip position should not perfectly follow its desired value and if it does, the walking loses the flexibility in the presence of the GRF impacts and other external effects.

Figure 5 shows the control signals of RAIC and RCAIC (the control force for the hip, and the control torques for the thigh and knee) with 30% parameter deviations. The control magnitudes for the off-nominal case have similar magnitudes as able-bodied averaged hip force (–800 to 200 N), thigh torque (–50 to 100 Nm), and knee torque (–50 to 50 Nm) [52–54]. Note that the hip force and thigh torque represent able-bodied walking, and the knee torque acts on the prosthesis, which has the same magnitude as able-body knee torque. This indicates a strong potential for the proposed controllers to be useful in real-world prosthesis applications. In addition, the results demonstrate that the controllers can deal with parameter variations without large increases in the control magnitudes.

For both controllers, high gains in the reference impedance model not only provide better tracking, particularly for hip displacement, but also increase the control effort. Figure 6 depicts the GRFs when the system parameters vary by 30% from nominal. We see that the generated forces are similar to able-bodied averaged horizontal GRF (–150 to 150 N) and vertical GRF (0–800 N) [52–54], again indicating strong potential for real-world application. As can be observed from Fig. 6, we have no GRF in swing phase, and after the point foot hits the ground (circles on the x-axis), horizontal and vertical GRFs become nonzero.

**4.4.2 Parameter Identification.** Figure 7 shows the estimated parameter vector $p$ (presented in Eq. (10)) for RAIC and RCAIC when the system parameters vary by 30%. As expected, the RAIC parameter estimates do not match the true parameter values.
However, RCAIC using the BGF composite adaptation law performs noticeably better regarding parameter estimation compared to RAIC. The estimated parameter vector of the proposed controller RCAIC perfectly matches the true value except for the fourth element $P_4$. $P_4$ is the most complex parameter in terms of its constituent elements (see Eq. (10)), so errors in the constituent elements of $P_4$ can cause cumulative errors in $P_4$.

Figure 9 compares the trajectories of $s$ and $s_3$ as described in Eqs. (13) and (24), respectively for RAIC and RCAIC. Based on the values of $q_1, q_2, q_3$, and the definition of $s_3$, it is seen that the TEB adaptation mechanism in RAIC is active only when $s$ is outside its boundary layer (i.e., $s_3$ is nonzero). When $s_3$ is zero, the parameter adaptation of the RAIC (Eq. (27)) stops and its estimated parameters remain constant, whereas $s_3 = 0$ only turns off TEB adaptation part of the RCAIC (the first part of the Eq. (29)).

It is observed that none of the $s$ trajectories for the RCAIC exceed the boundary layer (the area between $s_3 = 0.5$ and $s_3 = +0.5$) and in turn all $s_3$ trajectories are zero. This shows that the RCAIC only uses prediction errors, which appear in the TEB adaptation, and the TEB adaptation mechanism is turned off.

On the other hand, all $s$ trajectories for the RAIC exceed the boundary layer. From Fig. 8, we can see that the $s$ trajectories of the RAIC for the hip, thigh, and knee exceed the boundary layer four, three, and two times, respectively, and in turn, the $s_3$ trajectories are nonzero.

### 4.4.3 Persistent Excitation Verification

Figure 9 shows the norm of the adaptation gain $P$, the time-varying forgetting factor $\bar{v}(t)$, and the joint prediction errors ($e_\rho, i = 1, 2, 3$) for the RCAIC with $+30\%$ uncertainty on the system parameters. Figure 9(a) illustrates that $P(t)$ is upper and lower bounded by two positive numbers ($P(t)$ is upper bounded by $K_0 = 400$ and lower bounded by $P(0) = 100$). Figure 9(b) shows that the forgetting factor satisfies the condition $\bar{v}(t) > \bar{v} > 0$. These observations imply that $W(t)$ is PE. Since $W(t)$ is PE, $\bar{v}$ and $e_\rho$ exponentially converge to zero as shown in Fig. 9(c).

### 4.4.4 Numerical Evaluation

Table 4 summarizes the desired trajectory tracking, parameter estimation, and control performance for RAIC and RCAIC for the nominal system parameter values and also when the parameter values vary $\pm 30\%$ relative to nominal.

Table 4 lists total desired trajectory tracking cost $C_{t}^{E}$, total control cost $C_{t}^{u}$, total estimation cost $C_{t}^{p}$, and total cost $C_{t}$ (which is sum of the desired trajectory tracking and control costs) for both controllers. Table 4 shows that for the nominal case, RAIC has better performance for the control cost and estimation, while tracking performance maintains the same level as RCAIC, and in turn RAIC slightly improves the total cost by $1.2\%$. When the system parameter values vary $-30\%$, RCAIC has a small improvement in control cost, but an improvement in estimation by $40\%$ in comparison with the RAIC, while tracking performance of the RCAIC slightly deteriorates. In general, in the case of $-30\%$ parameter uncertainty, the total cost of the RCAIC decreases by $4\%$.

Table 4 shows that when the parameter values vary $30\%$ from nominal, RCAIC has a remarkable superiority to the RAIC in terms of the estimation and tracking performances. This superiority is because by more accurately estimating the system parameters, the RAIC includes a more accurate model ($\dot{p}$ and $e_\rho$ exponentially converge to zero) and in this way achieves better tracking. Table 4 shows that desired trajectory tracking performance ($C_{t}^{E}$) and estimation performance ($C_{t}^{p}$) of the proposed RCAIC considerably improves by $9.5\%$ and $76\%$, respectively, whereas the control signal magnitude ($C_{t}^{u}$) and total cost ($C_{t}$) increases by $9.9\%$ and $3.6\%$, respectively compared with RAIC. As it is seen from Table 4, the most notable difference between the controllers is their parameter estimation performances while their tracking performances are not noticeably different from each other. This is because the RCAIC uses a different adaptation law to improve the parameter estimation accuracy, whereas both controllers use the same control law.

Table 5 shows the maximum control effort ($U_{\text{max}}$), maximum tracking error ($E_{\text{max}}$), and maximum estimation error ($P_{\text{max}}$) for the nominal and off-nominal cases. The table shows that RCAIC results in smaller $U_{\text{max}}$ for the nominal and $+30\%$ cases, whereas RAIC performs better for the $-30\%$ case. Although RCAIC outperforms RAIC in terms of average parameter estimation for the off-nominal cases (Cost$_p$ in Table 4), their $P_{\text{max}}$ values are within approximately $1\%$ of each other. Table 5 also shows that $E_{\text{max}}$ is about the same for both controllers for the nominal and off-nominal cases.

### 5 Conclusions and Future Work

We designed two robust adaptive impedance controllers, RAIC and RCAIC, for a combined test robot and transfemoral prosthesis device. The controllers estimate the system parameters and also driving joint tracking errors to boundary layers while compensating for the variations of GRFs and nonparametric uncertainties. We defined the boundary layers to make a tradeoff between control signal chatter and performance, and also to stop TEB adaptation mechanism in these layers to prevent unfavorable parameter drift.

We designed both controllers to imitate the characteristics of natural walking and to provide flexible, smooth, gait. We thus defined a reference model with impedance similar to that of able-bodied gait. We also proved closed-loop system stability for both RAIC and RCAIC based on nonscalar boundary layers using Barbalat’s lemma and Lyapunov theory.
We performed simulations for both proposed controllers with 30% parameter errors, and we showed that trajectory tracking remained good, which demonstrated robustness of the proposed controllers. We demonstrated good transient responses with nominal system parameter values and also with system parameter value deviations of up to ±30%. When we used the first controller RAIC for the 30% parameter deviations, desired trajectory tracking errors were 16 mm for vertical hip position, 0.15 deg for thigh angle, and 0.12 deg for prothetic knee angle. When we used the second controller, RCAIC, with 30% parameter uncertainties, trajectory tracking errors were 14 mm for vertical hip position, 0.15 deg for thigh angle, and 0.08 deg for knee angle.

Therefore, numerical results showed that when the system parameter values varied by 30% from nominal, the proposed controller RCAIC had better tracking performance by 9.5% in comparison to RAIC, while resulting in more control cost by 9.9%. Furthermore, RCAIC using the BGF composite adaptation law achieved much better parameter estimation by 76% compared to the RAIC. We also achieved reasonable control signals and GRFs for the both controller structures. Note that, however, RAIC, in general, performed better than RAIC; RCAIC has larger computational time and higher programming complexity.

For future work, we will incorporate rotary and linear actuator dynamics in the system model to obtain motor voltage control signals. We will also apply the controllers to a prosthesis prototype that has been developed at Cleveland State University. We will also include an active ankle joint to the system model to extend the controllers to a 4-DOF robot/prosthesis model. We will test the proposed prosthesis model and controllers on a human-prosthesis hybrid system. We will also implement the proposed controllers experimentally on a powered transfemoral prosthesis, AMPRO3 (AMBER Prosthetic) [24]. In this paper, all system parameters are perturbed equally and at the same time, so a more in-depth sensitivity analysis of the system performance to the parameter variation would be interesting. The results of this paper are obtained for one experimental reference trajectory (Sec. 4.1). So, understanding the effects of varying speed, stride frequency, and other gait kinematic parameters on the overall performance will be an interesting problem.

We note that the inside the boundary layer in Eq. (26), \( \dot{s}_A = 0 \), and outside the boundary layer \( \dot{s}_A = s \), so using the closed-loop form in Eq. (25) gives

\[
V(s_A, \dot{s}) = s_A^T \left(-Cs - K_d \text{sat}(s/diag(\varphi)) + \left(\tilde{T}_e - T_e\right) + Y(q, \dot{q}, v, \dot{v})\right) + \frac{1}{2} \left(s_A^T M s_A + \dot{s}_A^T M \dot{s}_A\right) + \frac{1}{2} \left(p^T \mu \dot{p} + \dot{p}^T \mu p\right)
\]

(A2)

To derive the adaptation law, we constrain \( \dot{s}_A^T M \dot{s}_A \) and \( s_A^T Y(q, \dot{q}, v, \dot{v})p + p^T \mu \dot{p} \) to zero, which gives the update law \( \dot{\hat{p}} = -\mu^T Y(q, \dot{q}, v, \dot{v}) s_A \) as already presented in Eq. (27). As seen from Eq. (27), the adaptation law extracts information about the parameters from only the tracking error (i.e., TEB). Therefore, \( V(s_A, \hat{p}) \) can be written as follows:

\[
V(s_A, \hat{p}) = -s_A^T Cs + \frac{1}{2} \left(s_A^T M s_A\right) - s_A^T K_d \text{sat}(s/diag(\varphi)) + s_A^T \left(\tilde{T}_e - T_e\right)
\]

(A3)

We see from Eq. (26) that if \( |s| \leq \text{diag}(\varphi) \), then \( s_A = 0 \) and \( V(s_A, \hat{p}) \) converges to zero inside the boundary layer. Conversely, if \( |s| > \text{diag}(\varphi) \), then \( s_A \) is defined by the second part of Eq. (26), in which case \( s = s_A + \text{diag}(\varphi) / \text{diag}(\varphi) \) outside the boundary layer. If we substitute \( s = s_A + \text{diag}(\varphi) / \text{diag}(\varphi) \) in Eq. (57), we obtain \( V(s_A, \hat{p}) \) outside the boundary layer as follows:

\[
V(s_A, \hat{p}) = \frac{1}{2} s_A^T (M - 2C) s_A - s_A^T C \varphi \text{sat}(s/diag(\varphi)) - s_A^T K_d \text{sat}(s/diag(\varphi)) + s_A^T \left(\tilde{T}_e - T_e\right)
\]

(A4)

Matrix \((M - 2C)\) is skew symmetric, so \( s_A^T (M - 2C) s_A = 0 \) and we simplify \( V(s_A, \hat{p}) \) as

\[
V(s_A, \hat{p}) = -s_A^T (C \varphi + K_d) \text{sat}(s/diag(\varphi)) + s_A^T \left(\tilde{T}_e - T_e\right)
\]

(A5)

We choose \( K_d \) and \( \varphi \) as tuning parameters to keep \( C \varphi + K_d \) bounded from below by the \( K_d l \), where \( K_m \) is a positive scalar. We can see that \( C \varphi + K_d \geq K_d l \) ensures that \( C \varphi + K_d \) is positive definite. We use Eq. (A5) to write

\[
\dot{V}(s_A, \hat{p}) \leq -K_m \|s_A\|_1 + s_A^T \left(\tilde{T}_e - T_e\right)
\]

(A6)

We note that \( s_A^T \text{sat}(s/diag(\varphi)) \) is the one-norm of \( s_A \), so we write Eq. (A6) as

\[
\dot{V}(s_A, \hat{p}) \leq -K_m \|s_A\|_1 + s_A^T \left(\tilde{T}_e - T_e\right)
\]

(A7)

We now define \( K_m = F_m + \gamma_m \), where \( \|\tilde{T}_e - T_e\|_1 \leq F_1 \leq F_m \), \( F_m = \max(F_i) \), and \( \gamma_m = \max(\gamma_i) \). We can then write Eq. (A7) as follows:

\[
\dot{V}(s_A, \hat{p}) \leq -\gamma_m \|s_A\|_1 + F_m \|s_A\|_1 + s_A^T \left(\tilde{T}_e - T_e\right)
\]

(A8)

Noting that \( \|\tilde{T}_e - T_e\|_1 \leq F_1 \leq F_m \) and \( s_A \leq \|s_A\|_1 \), we see that \( s_A^T (\tilde{T}_e - T_e) \) in Eq. (A8) is bounded from above by \( F_m \|s_A\|_1 \), so

\[
\dot{V}(s_A, \hat{p}) \leq -\gamma_m \|s_A\|_1
\]

(A9)

This indicates that outside the boundary layer (the second condition of Eq. (26)), the Lyapunov derivative is negative.
semidefinite, so we can prove the stability of the closed-loop system with Barbalat’s lemma [50].

Barbalat’s Lemma: If a Lyapunov function $V = V(t, x)$ satisfies the following conditions:

(i) $V(t, x)$ is lower bounded, and
(ii) $V(t, x)$ is negative semi-definite, and
(iii) $\dot{V}(t, x)$ is bounded

then $\dot{V}(t, x) \to 0$ as $t \to \infty$; that is, the closed-loop system is asymptotically stable. Now we state an intermediate lemma that we will need to complete the proof of Theorem 1.

**Lemma 1.** The derivative of the Lyapunov function of Eq. (A9) converges to zero, which guarantees that the system converges to the boundary layer.

**Proof of Lemma 1:** Conditions I and II in Barbalat’s Lemma are confirmed from Eqs. (28) and (A9), which means that $V$ is bounded. This implies that all of the terms in $\dot{V}$ in Eq. (28) are bounded, including $\dot{s}$ and $\dot{p}$. Since $\dot{p}$ is constant, this means that $\ddot{p}$ is bounded. Since $s$ is bounded, this means that $\dot{s}$ is bounded. The second derivative of $V$ is bounded as follows: $\ddot{V}(s, \dot{p}) \leq -\gamma_m \frac{d}{dt} ||s||$.

In the worst case (i.e., at the upper bound), we have

$$\ddot{V}(s, \dot{p}) = -\gamma_m \frac{d}{dt} ||s|| = -\gamma_m \sum s_i \frac{d}{dt} ||s|| = \pm \gamma_m \sum s = \pm \gamma_m \sum s$$

(A10)

where $s \neq 0$ outside the boundary layer. We substitute $s$ from Eq. (25) into Eq. (A10) to obtain

$$\ddot{V}(s, \dot{p}) = -\gamma_m \sum s_i M^T (-C \xi - K_\gamma \text{sat}(s/diag(\phi))) + (\dot{T}_e - T_e) - Y(q, q, v, v)\ddot{p}$$

(A11)

Recall that $\dot{p}$ and $\ddot{p}$ are bounded. The boundedness of $s$ implies the boundedness of $\epsilon$ and $e$, as seen from Eq. (13). Since $q, q_1, q_2$, and $q_3$ are bounded, we know that $q, q_1, q_2, and q_3$ are also bounded. Therefore, in Eq. (A11), $M, C, \varphi, p, \dot{p}, \gamma_p, s$, and $K_\gamma$ are bounded. $\dot{T}_e - T_e$ is upper bounded by $F_m$, so we can conclude that $V$ is bounded. Therefore, as conditions I, II, and III from Barbalat’s Lemma hold, we can conclude that $\dot{V}(s, \dot{p}) \to 0$ as $t \to \infty$. This means that $-\gamma_m ||s||$, in Eq. (A9) is equal to zero, which means that Eq. (A9) can be written as the equality $\ddot{V}(s, \dot{p}) = -\gamma_m ||s||$. We, therefore, have $\ddot{V}(s, \dot{p}) \to 0 \Rightarrow -\gamma_m ||s|| \to 0 \Rightarrow s \to 0$. This indicates that the control ensures that $s$ converges to the boundary layer.

**A.2 QED (Lemma 1).** The RAIC mitigates system uncertainties more than a standard adaptive controller but also has a larger tracking error. RAIC drives the system to the boundary layer and results in robustness to GRF as a nonparametric uncertainty. Inside the boundary layer, Eqs. (A4)-(A11) can be reformulated for $s = 0$, in which case $s$ remains in the boundary layer, which stops adaptation, and the estimated parameters remain constant. Therefore, the system with the RAIC converges to the reference impedance model.

**A.3 QED (Theorem 1).** It should be noted that the aforementioned proof of asymptotic closed-loop system stability implies that $K_d$ should be driven by $F_m - C \phi$—that is, $K_d \geq F_m I_{3 \times 3} - C \phi + \gamma_m I_{3 \times 3}$.

**Appendix B**

**B.1 Stability Analysis of the Robust Composite Adaptive Impedance Controller. Proof of Theorem 2:** Note that $V$ in Eq. (42), which is a quadratic function of $s$, is continuously differentiable. The derivative of the Lyapunov function is given as

$$V(s, \dot{p}) = \frac{1}{2} (s_i^T M \dot{s}_i + s_i^T M \dot{s}_i) + \frac{1}{2} (\dot{s}_i^T M s_i) + \frac{1}{2} \left( \dot{s}_i^T P^{-1} \dot{p} + \dot{p}^T P^{-1} \dot{p} \right) + \frac{1}{2} \left( \dot{p}^T \frac{d}{dt} (p^{-1}) \dot{p} \right)$$

(B1)

Now we want to prove global exponential stability of the closed-loop system both outside and inside the boundary layer defined in Eq. (26). Inside the boundary layer $\dot{s} = 0$, and outside the boundary layer $\dot{s} = 0$, so Eq. (B1) can be written as

$$V(s, \dot{p}) = \frac{1}{2} (s_i^T M \dot{s}_i + s_i^T M \dot{s}_i) + \frac{1}{2} \left( \dot{s}_i^T P^{-1} \dot{p} + \dot{p}^T P^{-1} \dot{p} \right) + \frac{1}{2} \left( \dot{p}^T \frac{d}{dt} (p^{-1}) \dot{p} \right)$$

(B2)

Outside the boundary layer we see that if $|s| > diag(\phi)$, then $s_\alpha$ comes from the second condition of Eq. (26), in which case we have $s = s_\alpha + qast(\text{diag}(\phi))$. Substituting $s = s_\alpha + qast(\text{diag}(\phi))$ in the first term of Eq. (B2), we write $V(s, \dot{p})$ outside the boundary layer as

$$V(s, \dot{p}) = \frac{1}{2} (s_\alpha^T (M - 2C) s_\alpha - s_\alpha^T C qast(\text{diag}(\phi)) + s_\alpha^T Y(q, q, v, v) \ddot{p} + s_\alpha^T K_\gamma \text{sat}(s/diag(\phi)) + s_\alpha^T (\dot{T}_e - T_e) + \dot{p}^T P^{-1} \ddot{p} + \frac{1}{2} \left( \dot{p}^T \frac{d}{dt} (p^{-1}) \ddot{p} \right)$$

(B3)

$$(M - 2C)$$ is skew-symmetric, so $s_\alpha^T (M - 2C) s_\alpha = 0$ and we can simplify $V(s, \dot{p})$ as

$$V(s, \dot{p}) = -s_\alpha^T (C \phi + K_\gamma) \text{sat}(s/diag(\phi)) + s_\alpha^T Y(q, q, v, v) \ddot{p} + s_\alpha^T (\dot{T}_e - T_e) + \dot{p}^T P^{-1} \ddot{p} + \frac{1}{2} \left( \dot{p}^T \frac{d}{dt} (p^{-1}) \ddot{p} \right)$$

(B4)

We tune the design parameters $K_\gamma$ and $\varphi$ so that $C \phi + K_\gamma \geq M \phi$, which guarantees the positive definiteness of $C \phi + K_\gamma$, where $M$ is a positive scalar. We then use Eq. (B4) to write

$$V(s, \dot{p}) \leq -K_s s_\alpha^T \text{sat}(s/diag(\phi)) + s_\alpha^T Y(q, q, v, v) \ddot{p} + s_\alpha^T (\dot{T}_e - T_e) + \dot{p}^T P^{-1} \ddot{p} + \frac{1}{2} \left( \dot{p}^T \frac{d}{dt} (p^{-1}) \ddot{p} \right)$$

(B5)

We replace $s_\alpha^T \text{sat}(s/diag(\phi))$ with the one-norm of $s_\alpha$ and then we write Eq. (B5) as

$$V(s, \dot{p}) \leq -K_m ||s|| + s_\alpha^T Y(q, q, v, v) \ddot{p} + s_\alpha^T (\dot{T}_e - T_e) + \dot{p}^T P^{-1} \ddot{p} + \frac{1}{2} \left( \dot{p}^T \frac{d}{dt} (p^{-1}) \ddot{p} \right)$$

(B6)

We define $K_m = F_m + \gamma_m$, where $|\dot{T}_e - T_e| \leq F_l \leq F_m$, $F_m = \max(F_i)$, and $\gamma_m = \max(\gamma_i)$ with $i = 1, 2, 3$. Noting that $s_\alpha \leq ||s||$, we see that $s_\alpha^T (\dot{T}_e - T_e)$ in Eq. (B6) is bounded from above by $F_m ||s||$, so
\[
V(s_{\lambda}, \tilde{p}) \leq -\gamma_{m} \|s_{\lambda}\|_{1} + s_{\lambda}^{T} Y(q, \dot{q}, v, \dot{v}) \tilde{p} + \frac{p^{T} P^{-1} p}{2} + \frac{1}{2} \left( p^{T} D (P^{-1}) p \right)
\] (B7)

Since \( \tilde{p} = \tilde{p} - p \), we can substitute Eq. (29) and Eq. (36) into Eq. (B7), and \( V(s_{\lambda}, \tilde{p}) \) can be written as
\[
\begin{align*}
V(s_{\lambda}, \tilde{p}) &\leq -\gamma_{m} \|s_{\lambda}\|_{1} + s_{\lambda}^{T} Y(q, \dot{q}, v, \dot{v}) \tilde{p} \\
&+ \frac{1}{2} \left( p^{T} D (P^{-1}) p \right) - p^{T} Y(q, \dot{q}, v, \dot{v}) s_{\lambda} \\
&- \tilde{p}^{T} W^{T} R W \tilde{p} = -\gamma_{m} \|s_{\lambda}\|_{1} + \frac{1}{2} \left( p^{T} D (P^{-1}) p \right) \\
&- \tilde{p}^{T} W^{T} R W \tilde{p}
\end{align*}
\] (B8)

By substituting Eq. (37) into Eq. (B8), we can write
\[
V(s_{\lambda}, \tilde{p}) \leq -\gamma_{m} \|s_{\lambda}\|_{1} - \frac{1}{2} \left( p^{T} \dot{v}(t) (P^{-1} p - p^{T} W^{T} \left( dl - \frac{1}{2} I \right) W \tilde{p} \\
\right.
\] (B9)

where \( \gamma_{m} > 0, \dot{v}(t) \geq 0 \), and \( P \) is positive definite, so by choosing \( d > \frac{1}{2} \), we can see that outside the boundary layer (i.e., the second condition of Eq. (26)), the derivative of the Lyapunov function is negative semidefinite. This in turn means that we can use Barbalat’s Lemma to prove global exponential stability. If \( V(t, s) \) satisfies the Barbalat’s Lemma conditions, then \( V(t, s) \) \( \to 0 \) as \( t \to \infty \), which means that RCIAC results in a closed-loop system that is globally exponentially stable.

Now we state an intermediate lemma that we will need to complete the proof of Theorem 2.

**Lemma 2**: The derivative of the Lyapunov function of Eq. (B9) globally exponentially converges to zero, which guarantees convergence to the boundary layer \( (s_{\lambda} \to 0) \). Also, the prediction error in Eq. (36) of the proposed RCIAC converges to zero, which implies perfect estimation of the system parameters.

**Proof of Lemma 2**: Conditions I and II in Barbalat’s Lemma are satisfied from Eqs. (42) and (B9) and we therefore, conclude that \( V \) is bounded, which means that all terms in \( V \) (including \( s_{\lambda} \) and \( \tilde{p} \)) are bounded. Since \( p \) is the constant \( \tilde{p} \) is bounded, and since \( s_{\lambda} \) is bounded \( s \) is bounded. From Eq. (11), since \( \dot{q}_{s} \) is bounded, \( q_{s}, \dot{q}_{s}, q_{d}, \dot{q}_{d}, \) and \( \ddot{q} \) are bounded. From Eqs. (15)-(15), since \( s \) is bounded, we see that \( \dot{e} \) and \( e \) are both bounded. These facts imply that \( q, \dot{q}, \dot{q}_{s}, v, \dot{v} \) are bounded as well. So, \( Y(q, \dot{q}, v, \dot{v}) \), \( Y(q, \dot{q}, \dot{q}) \), and \( W(q, \dot{q}, \dot{q}) \) are bounded.

By taking the derivative of \( V(s_{\lambda}, \tilde{p}) \) at its upper bound, we obtain
\[
\begin{align*}
\dot{V}(s_{\lambda}, \tilde{p}) = \pm \gamma_{m} \sum_{\lambda} \dot{s}_{\lambda} - \tilde{p}^{T} W^{T} (2dl - l) \tilde{p} \\
- \tilde{p}^{T} W^{T} (2dl - l) W \tilde{p} - \tilde{p}^{T} \dot{v}(t) (P^{-1} p - \frac{1}{2} \tilde{p}^{T} \dot{v}(t) \frac{d}{dt} (P^{-1}) p)
\end{align*}
\] (B10)

Substituting \( \dot{s}_{\lambda} \), \( \frac{d}{dt} (P^{-1}) p \), and \( \tilde{s}_{\lambda} \) from Eqs. (29), (37), and (25), respectively, into Eq. (B10), \( V(s_{\lambda}, \tilde{p}) \) can be written as follows:
\[
\begin{align*}
\dot{V}(s_{\lambda}, \tilde{p}) = \pm \gamma_{m} \sum_{\lambda} M^{-1} (-C - K_{s} \text{sat}(s/diaq(\phi))) \\
+ (\bar{T}_{e} - T_{e}) Y(q, \dot{q}, v, \dot{v}) \tilde{p} + \tilde{p}^{T} W^{T} (2dl - l) W P(t) Y^{T} s_{\lambda} + \tilde{p}^{T} \dot{v}(t) Y^{T} s_{\lambda} + \tilde{p}^{T} W^{T} (2dl - l) W P(t) W^{T} (dl) W \tilde{p} - \tilde{p}^{T} W^{T} (2dl - l) W \tilde{p} + \tilde{p}^{T} \dot{v}(t) W^{T} (dl) W \tilde{p} - \frac{1}{2} \tilde{p}^{T} \dot{v}(t) (P^{-1}) p \\
+ \frac{1}{2} \tilde{p}^{T} \dot{v}(t) (P^{-1}) p - \frac{1}{2} \tilde{p}^{T} \dot{v}(t) W^{T} W \tilde{p}
\end{align*}
\] (B11)

Since \( P(t) \) is bounded and its norm is bounded by \( K_{0} \), then from Eq. (38), \( \dot{v}(t) \) and \( \ddot{v}(t) \) are bounded. Moreover, since \( M, C, Y, W, W, \tilde{p}, \) and \( s_{\lambda} \) are bounded and \( \bar{T}_{e} - T_{e} \leq F_{m} \), we see that \( \dot{V}(s_{\lambda}, \tilde{p}) \) is bounded. Since we have verified all conditions in Barbalat’s Lemma, we know that \( \dot{V}(s_{\lambda}, \tilde{p}) \to 0 \) as \( t \to \infty \).

**B.3 QED (Theorem 2)**

We see from the above that the closed-loop system with the proposed RCIAC converges to the target impedance model. Therefore, the controller drives the system to the boundary layer, achieves perfect parameter estimation, and achieves robustness against GRFs.

**References**


