Application of GPS for Missile Post Flight Guidance Accuracy Analysis

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Abstract

The high quality of a GPS-based trajectory reference makes it useful for evaluating the performance of a missile's Inertial Navigation System (INS) during a missile flight test. The complexity of the design of the GPS user segment depends on flight test objectives and factors such as the expected satellite/vehicle geometry. Design curves are developed which illustrate the relationship between flight test objectives, the GPS user segment design, and such factors as antenna performance.

I. Introduction

Because of the versatility provided by its global availability and the passive nature of the user segment, the Global Positioning System (GPS) is being used in a wide range of aerospace applications [1][2]. Among these is testing inertial navigation systems.

GPS user segment designs can be broadly classified into two categories: receiver- and translator-based designs. A GPS receiver processes GPS signals to estimate its own position and velocity. It must process the signals and compensate for known measurement errors in real time. A GPS translator, on the other hand, is a relatively simple device whose function is to frequency shift ("translate") the GPS signals from one frequency band to another, such as a telemetry band. The translated signal is then retransmitted to a ground receiving station, where it is time-tagged and processed or recorded for later processing.

Translator-based user segments offer several advantages, including low cost, weight, and power consumption and high reliability. Furthermore, ground post-processing of the signals allows for the use of highly accurate satellite orbital information not available in real-time and highly detailed corrections. It also allows analysts to iteratively edit the data and respond to anomalous conditions. The result is accuracy better than that achievable by a receiver doing real-time navigation.

Recent United States Air Force (USAF) flight testing has demonstrated that a GPS translator can provide a small, light, affordable, and accurate trajectory reference for flight testing land-based missiles [3]. Some of the technology used had been demonstrated previously by the United States Navy during Trident missile testing [4]. The primary use of the trajectory reference is for post flight evaluation of errors in the missile's inertial navigation system (INS).

This paper describes factors that influence the design of a GPS translator used as a trajectory reference for the post flight evaluation of a missile's INS.

II. GPS Translator Design

A. Contributors To System Performance

The ability to meet flight test objectives depends on the design of the GPS user segment as well as other factors relating to the system performance [5]. This relationship is illustrated in Figure 1. Each box represents either a measure of performance or a factor affecting performance; boxes higher in the figure depend on boxes connected to them from below.

At the top of the figure is the user's requirement on the performance of the whole system. For a GPS translator used as a trajectory reference for flight testing inertial navigation systems, the user requirement is on measures of the ability to estimate INS errors.

The three-dimensional measurement accuracy of the GPS user segment can be determined independently of other components in the user's system. It depends on the satellite geometry [6][7], the vehicle flight path, and the one-dimensional GPS measurement accuracy [8][9]. The measurement accuracy depends on the receiver or translator design, the antenna design, the accuracy of the satellite ephemeris data, relativity and atmospheric effects, and fixed characteristics of GPS.

Translators can be designed to process the L\textsubscript{1} (1575.42 MHz) or L\textsubscript{2} (1227.60 MHz) signals or both. Processing two frequencies allows for better ionospheric refraction corrections. In addition, translators can be designed to process one or both of the GPS codes. The GPS L\textsubscript{1}
The signal is quadrature modulated by two pseudorandom codes, a 1.023 Mbit/s coarse acquisition (C/A) code and a 10.23 Mbit/s precision (P) code [10][11]. The type of code used determines the range precision that can be achieved.

The design and calibration of the antenna affects the accuracy of the phase-derived delta range measurement [12]. The antenna phase induces a delta range error through three mechanisms: error in the phase calibration, vehicle attitude error coupled with the antenna phase slope, and ionospheric refraction correction error. The accuracy of the phase center calibration also affects the calculation of vehicle reference point to phase center lever arm, effectively introducing measurement errors.

The GPS satellite ephemerides are obtained either in real time from the GPS navigation message [13] or from satellite tracking data spanning a period of several days both before and after the time of interest.

Two different data correction schemes are considered. The coarser correction scheme adjusts the GPS measurements for satellite clock phase and frequency, drift in the translator carrier frequency, and changes in the signal path length due to ionospheric and tropospheric refraction. A coarse correction for relativistic effects is also built into the GPS clock frequency. A finer approach does the coarse corrections plus precise corrections for general and special relativistic effects due to the vehicle motion and higher accuracy tropospheric refraction corrections based on weather data.

Fixed characteristics of GPS include the satellite clock phase and frequency accuracy after correction. Contributing to the GPS delta range resolution are the carrier wavelength, errors in the phase tracking loop, and atmospheric effects.

B. GPS Error Model

The various contributors to GPS measurement errors were modeled and then simulated to assess their impact on the user segment performance. Although this model applies to a translator-based user segment, it can be used for receivers by taking the receive time and location to be coincident with the translation time and location.

The GPS range measurement is modeled as

\[
R'(t_k) = r'(t_k) + S'(t_k) C_{RH}(t_k) \cdot X_r + (t_k - t_c) \cdot S'(t_k) C_{RH}(t_k) \cdot X_r + \frac{c}{10^6} \cdot X_{CF} + B'(t_k) X_{TSF}(t_k) + X_{RAI}(t_k) + S'(t_k) C_{RA}(t_k) \cdot X_{LA} + v_r(t_k)
\]

where

- \( R' \) is the measured range from the \( i \)th satellite to the vehicle to the ground;
- \( r' \) is the true range;
- \( t_c \) is the ground receive time;
- \( t_v \) is the vehicle translation time;
- \( t_t \) is the satellite transmission time;
- \( S' \) is the unit vector from the vehicle to the \( i \)th satellite;
- \( C_{RH} \) is the direction cosine matrix from the satellite frame to the \( i \)th satellite to the reference frame;
- \( C_{RA} \) is the direction cosine matrix from the vehicle body frame to the reference frame;
- \( c \) is the speed of light;
- \( B' \) is the tropospheric refraction correction for the \( i \)th satellite;
- \( X_{CF}, X_{LA}, X_{TSF}, X_{RAI} \), and \( X_{RA} \) are per-satellite GPS errors: satellite position, satellite velocity, clock frequency, clock phase, tropospheric scale factor, and residual ionospheric refraction;
- \( X_{LA} \) are global GPS errors: antenna lever arms;
- \( v_r \) is the range measurement noise for the \( i \)th satellite.

The GPS delta range measurement is modeled as follows:

\[
D'(t_k) = d'(t_k) + [S'(t_k) C_{RH}(t_v) \cdot S'(t_v) C_{RH}(t_v)] X_r + [(t_v - t_c) S'(t_v) C_{RH}(t_v)] X_r + (t_k - t_v) X_{CF} + B'(t_k) X_{TSF}(t_k) - B'(t_k) X_{TSF}(t_k) + X_{DRI}(t_k) - X_{DRI}(t_k) + X_{DRA}(t_k) - X_{DRA}(t_k) + [S'(t_k) C_{RH}(t_k) \cdot S'(t_k) C_{RH}(t_k)] X_{LA} + X_{GR}(t_k) + X_{SR}(t_k) + v_{AC}(t_k) - v_{AC}(t_k)
\]

where

- \( D'(t_k) \) is the measured delta range from the \( i \)th satellite to the vehicle to the ground over the interval \((t_{k-1}, t_k)\);
- \( d'(t_k) \) is the true delta range;
- \( X_{DRI} \) and \( X_{DRA} \) are per-satellite GPS errors: Doppler residual ionospheric refraction and antenna phase;
- \( X_{GR} \) and \( X_{SR} \) are residual general and special relativity effects;
- \( v_{AC}(t_k) - v_{AC}(t_k) \) is the one-step anticorrelated delta range measurement noise for the \( i \)th satellite; \( v_{AC}(t_k) \) is white and Gaussian.

The GPS errors \( X_i \) and \( X_n \) (where \( n = \text{TSF, LA, etc.} \)) are assumed to be constants, random constants, or
random variables from a first-order Gauss-Markov process.

C. Flight Testing Inertial Navigation Systems

Estimating the sources of the INS error requires a separate trajectory reference. In the past, ground-based radars and, sometimes, a second on-board INS have been used to provide the reference. The second option is usually prohibitive in terms of both cost and payload restrictions, while radars suffer from limitations in both geometry and accuracy. GPS translators can provide a cheap and accurate trajectory reference for evaluating INS errors.

To estimate INS errors the INS telemetry is processed with the GPS measurement data to generate observations that are functions of the INS and GPS errors. Specifically, the corrected GPS range and delta range are differenced with the equivalent quantities as indicated by the INS under test. These INS-indicated ranges and delta ranges are computed using integrated accelerometer data and the satellite ephemerides. The GPS minus INS-indicated ranges and delta ranges are used as the observations for a Kalman filter [14]. The Kalman filter state vector contains an element for each modeled INS and GPS error.

The observation matrix for the Kalman filter is derived from error models for accelerometers, gyros, initial conditions, and GPS. These models are combined with differential equations for the propagation of errors in an inertial navigation system to produce the sensitivities of the filter observations to INS and GPS errors.

D. Performance Measures

The performance of a trajectory reference during flight testing can be quantified using various measures. One important measure is the total estimation uncertainty, produced by the filter in the form of a large covariance matrix, a square matrix with a side dimension equal to that of the filter state. Because the uncertainty in this form is very unwieldy, a preferable measure is a circular error probable (CEP) based on it. This scalar is produced by propagating the state space error covariance matrix into impact space and then calculating a 50th percentile radius.

Other measures of performance include the group estimation uncertainties, also indicated by CEPs. These quantities are calculated in a fashion similar to the total estimation uncertainty CEP, except they are based only on certain submatrices of the error covariance matrix corresponding to the accelerometer, gyro, or initial condition (including INS clock) error groups.

Performance can also be measured by the individual error state recovery ratios. The recovery ratio for an error state is the final standard deviation of a state estimate divided by the initial standard deviation. It represents the ratio of final uncertainty to initial uncertainty for an individual error state; low recovery ratios indicate better estimates. Rather than tabulate the recovery ratios for every error state, the measure used is the fraction of states having a recovery ratio exceeding 0.5.

III. GPS User Segment Design Analysis

Four fundamental design parameters were varied in this analysis: the code type (C/A or P), the number of frequencies (single or dual), the measurement correction scheme (coarse or fine), and the antenna phase calibration error standard deviation (from 20 to 80 degrees in ten degree increments). Four user segment configurations, representing various combinations of code type, frequency usage, and measurement correction scheme, were studied. Furthermore, the antenna phase calibration error was varied for one of the configurations. The analysis considered a missile on a 4000 mile trajectory and nominal satellite coverage.

The INS model for this study contained a total of 76 terms, including 33 accelerometer terms, 33 gyro terms, nine initial condition terms, and one IMU clock frequency error (grouped with the initial condition terms for convenience). The INS errors are modeled as being initially random (Gaussian with zero mean and some assumed standard deviation) but constant throughout the flight; the effects of the errors on the navigated state and the GPS observations are functions of time. Three different classes of INs were considered, representing systems with 500, 5000, and 50000 foot CEPs.

Figure 2 shows the effects of the user segment configuration on the ability to estimate the total INS navigation error. The largest effects on estimation uncertainty are the code type, which establishes the range resolution, and whether a second frequency is used for ionospheric corrections. The antenna phase calibration error also has a significant effect for the dual frequency configurations; otherwise, its contribution gets swamped by the ionospheric refraction error.

The ability to estimate the errors of different INs, for a configuration using P code, dual frequencies, and fine corrections, is illustrated in Figures 3 through 7. Note
Estimation uncertainty for the total INS error is good for all three classes of INSs, but is dependent on antenna phase calibration error. On the other hand, the ability to estimate major INS error groups is relatively limited, especially for less accurate INSs (Class III). Even for the most accurate INSs (Class I), the estimation uncertainty for the groups is several times higher than for the total. This indicates that GPS is able to estimate the total INS performance quite well, but the estimation errors for the INS error groups are highly correlated to one another.

Finally, GPS is seen to be limited in its ability to discern individual INS errors, as seen in Figure 7. Only a third to a half of the individual errors have estimation uncertainties significantly less (i.e., 50%) than the a priori uncertainty. Note that more individual errors can be recovered from less accurate INSs, where the effects of individual errors are larger.

IV. Summary

The results of this study indicate that the ability to track P code on two different frequencies is the most critical aspect of GPS user segment design. In addition, the extra effort required to perform fine data corrections, especially tropospheric refraction corrections, results in a significant improvement in GPS accuracy. Antenna phase calibration is critical only if dual frequency tracking is used.

A GPS-based trajectory reference system allows for good estimation of the total INS error. Estimates of grouped and individual INS errors have more uncertainty.

References


Figure 1 - User Segment Design