A PASSIVITY-BASED REGRESSOR-FREE ADAPTIVE CONTROLLER FOR ROBOT MANIPULATORS WITH COMBINED REGRESSOR/PARAMETER ESTIMATION

Donald Ebeigbe  
Department of Electrical Engineering  
and Computer Science  
Cleveland State University  
Cleveland, Ohio 44115  
Email: d.ebeigbe@csuohio.edu

Dan Simon  
Department of Electrical Engineering  
and Computer Science  
Cleveland State University  
Cleveland, Ohio 44115  
Email: d.j.simon@csuohio.edu

ABSTRACT

This paper develops a new function approximation technique (FAT)-based adaptive controller for the control of rigid robots called the adaptive passivity function approximation technique (APFAT) controller. This controller utilizes the passivity-based approach and simplifies the FAT controller design by eliminating the need for simultaneous estimation of the robot's inertia matrix, Coriolis matrix, and gravity vector. The controller achieves its simplicity by treating the product of the regressor matrix and parameter vector as an unknown time-varying function to be approximated. The controller can be implemented in robots where the dynamic equations of motion are unknown. The stability of the controller is verified with Lyapunov functions by taking advantage of the passivity property of the robot dynamics. Simulation results on a three degree-of-freedom (DOF) PUMA500 robot demonstrate the ability to track reference trajectories using reasonable control signals when the inertia matrix, Coriolis matrix, and gravity vector are unavailable.

1 INTRODUCTION

Adaptive control has received significant attention from researchers and has been implemented in practical systems for several decades because it can handle system uncertainties that vary with time. The adaptive inverse dynamics approach [1, 2] uses the inverse of the estimate of the inertia matrix. The drawback of this method is that although the inertia matrix is nonsingular, its estimate may be singular, thereby leading to numerical problems. Some adaptive control schemes use joint acceleration feedback [2–4], which might prove difficult or costly to measure and which might be noisy. The assumption that the joint acceleration is available and that the estimated inertia matrix remains nonsingular is used in the controller development of [2]. The requirement that the estimated inertia matrix remains nonsingular was relaxed in [4]. Another method that eliminates the need for the invertibility of the estimated inertia matrix and the availability of joint acceleration feedback was developed by restricting the steady-state position errors to lie on a sliding surface [5].

A linear parameterization of the dynamic equations of motion of a robot includes a regressor matrix and a parameter vector. Regressor-based adaptive control has gained traction in recent decades [6–8]. In [6], the variable structure control scheme was used to develop an adaptive controller for underactuated robots. In [7], adaptive model reference impedance controllers were developed for the control of robots interacting with humans. A regressor-based blend of adaptive control with robust control was used to derive a stable controller in the presence of external forces [8]. For any robot, parameter vectors with various dimensions can be found, but a minimal parameterization can always be found [9, 10]. A regressor-based controller that utilizes filtered forms of the regressor matrix was designed to be robust to disturbances [11, 12].

Robot models have the passivity property and several control techniques take advantage of this property [3, 6–8, 12–14]. Passivity-based control was first introduced in [13] to make the closed loop system passive. In [12], a robust composite adaptive
controller was developed using bounded-gain forgetting. A passive least squares type estimation algorithm was used to develop an adaptive controller for robot manipulators in [14]. The passivity framework was used to develop an impedance controller for a transfemoral prosthetic leg that included both pure motion tracking and force interaction control [15].

In contrast to regressor-based control, several control methods forgo the use of the regressor matrix [16, 17]. The function approximation technique (FAT) controller is a regressor-free approach for the control of robots [17–21]. The FAT controller design uses the premise that the robot’s dynamic equation is unavailable. The FAT controller uses a finite linear combination of orthonormal basis functions to estimate the dynamic equations of a robot [19]. Several FAT controllers [17, 18, 21] are based on Slotine and Li’s method [5], which eliminates the need for joint acceleration feedback. The FAT controller and a regressor-based controller were combined to yield a stable hybrid controller with good performance in the presence of parametric uncertainties [21].

The use of more basis functions in the FAT controller leads to increased matrix dimensions [18], which in turn leads to additional complexity, computational effort, and memory requirements. Several FAT-based controllers have been developed with the aim of simplicity in controller design and implementation [22, 23]. In [22], a controller is developed for the control of an electrically driven robot by using a simple FAT-based voltage control strategy. A state feedback FAT controller was developed for an electrically driven robot in [23]. In [20], the joint acceleration vector and the robot dynamics were lumped together to develop a simplified adaptive FAT-based controller.

The main contribution of this paper is the development of a new adaptive FAT controller called the adaptive passivity function approximation technique (APFAT) controller. The APFAT controller simplifies the FAT controller [17–19] by applying the passivity property. The new control method in this paper offers simplicity in controller design and implementation by eliminating the need for simultaneous approximation of the robot’s inertia matrix, Coriolis matrix, and gravity vector, while still maintaining stability and performance. The controller simplicity is achieved by approximating an unknown time-varying function, which is defined as the combined product of the regressor matrix and the parameter vector. The goal of the controller is to achieve pure motion tracking of a predetermined trajectory. When compared to the simplified adaptive FAT controllers of [20, 22–24], our controller is advantageous by having fewer tuning parameters. A Lyapunov stability analysis of the controller ensures uniform ultimate boundedness. The performance of the controller is verified by simulation on a three degree-of-freedom (DOF) PUMA500 robot with a completely unknown inertia matrix, Coriolis matrix, and gravity vector.

This paper is organized as follows. Section 2 discusses the dynamic equation of a robot and reviews some existing techniques. Section 3 develops the new APFAT controller for robots with unknown dynamics and proves its stability via a Lyapunov function. Section 4 validates the APFAT controller via computer simulations. Section 5 gives the conclusion and a discussion of future work.

2 Dynamic System Description and Review of Existing Techniques

This section gives a brief description of the dynamic system, along with a review of the adaptive passivity (AP) controller [25], and the adaptive function approximation technique (FAT) controller without Slotine and Li’s modification [20].

2.1 Dynamic System

The motion of an n-degree-of-freedom (DOF) rigid robot without external forces can be described by the dynamic equation

\[ D(q)\ddot{q} + C(q, \dot{q}) \dot{q} + g(q) = \tau \]  

(1)

where \( q \in \mathbb{R}^n \) is the vector of generalized joint displacements, \( D(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is the matrix of Coriolis and centripetal forces, \( g(q) \in \mathbb{R}^n \) is the gravity vector, \( \tau \in \mathbb{R}^n \) denotes the torque input. We note the following properties of Eqn. (1), which will be used in subsequent sections.

Property 1. The inertia matrix \( D(q) \) is a positive definite matrix with eigenvalues that satisfy \( 0 < \lambda_1(q) \leq \lambda_2(q) \leq \lambda_3(q) \cdots \leq \lambda_n(q) \) such that \( \lambda_1(q)I_n \leq D(q) \leq \lambda_n(q)I_n \), where \( I_n \) is the \( n \times n \) identity matrix.

Property 2. The matrix \( D(q) - 2C(q, \dot{q}) \) is skew-symmetric.

Property 3. The left-hand side of Eqn. (1) can be linearly parameterized in the form

\[ Y_r(q, \dot{q}, \ddot{q}) \theta = \tau \]  

(2)

where \( Y_r(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{n \times l} \) is the regressor matrix, \( \theta \in \mathbb{R}^l \) is a vector of parameters, and \( l \) is the number of parameters, which is not unique, but a minimal parameterization can always be found.

In the sequel, the notation for \( D, C, \) and \( g \) will not explicitly indicate dependence on \( q \), for ease of notation.

2.2 Adaptive Regressor-Based Control

This section reviews the adaptive passivity (AP) controller, which is a regressor-based controller [25]. We define

\[ v = \dot{q} - \Lambda \ddot{q} \]  

(3)

\[ a = v - \dot{q} - \Lambda \ddot{q} \]  

(4)

\[ r = q - v = \dot{q} + \Lambda \ddot{q} \]  

(5)
where \( \Lambda \in \mathbb{R}^{n \times n} \) is a tunable diagonal matrix with positive diagonal entries and \( q_d \in \mathbb{R}^n \) is the reference trajectory. Using Eqns. (3), (4), and (5), we rewrite Eqn. (1) as

\[
Dr + Cr + Da + Cv + g = \tau \tag{6}
\]

Using Property 3, Eqn. (6) becomes

\[
Dr + Cr + Y(q, q, \dot{q}, a)\theta = \tau \tag{7}
\]

where \( Y(q, q, \dot{q}, a) \in \mathbb{R}^{n \times l} \) is the regressor matrix that is independent of joint accelerations. We will simply write \( Y \) in place of \( Y(q, q, \dot{q}, a) \) in the sequel for ease of notation. The control law is given as

\[
\tau = Y\hat{\theta} - Kr \tag{8}
\]

where \( \hat{\theta} \in \mathbb{R}^l \) is the estimate of the parameter vector \( \theta \), and \( K \in \mathbb{R}^{n \times n} \) is a tunable diagonal matrix with positive diagonal entries. Substituting Eqn. (8) into Eqn. (7) gives the closed loop dynamics

\[
D\ddot{r} + Cr + Y\theta = Y\hat{\theta} - Kr \tag{9}
\]

The estimate \( \hat{\theta} \) is obtained by the update law

\[
\dot{\hat{\theta}} = -\Gamma^{-1}Y^T r \tag{10}
\]

where \( \Gamma \in \mathbb{R}^{l \times l} \) is a tunable diagonal matrix with positive diagonal entries.

**Theorem 1.** Using the update law of Eqn. (10), the closed loop dynamics of Eqn. (9) is asymptotically stable.

*Proof. See [25].*

### 2.3 Adaptive Regressor-Free Control without Slotine and Li's Modification

This section reviews an adaptive regressor-free controller that improves on the adaptive FAT controller design in [17] by simplifying the controller structure [20]. This is done without Slotine and Li's modifications. We rewrite Eqn. (1) as

\[
D\ddot{q} + C\dot{q} + g - \ddot{q} + \ddot{q} = \tau \tag{11}
\]

Defining an unknown time-varying vector \( \Psi(t) \in \mathbb{R}^n \) as

\[
\Psi(t) = D\ddot{q} + C\dot{q} + g - \ddot{q} \tag{12}
\]

we simplify Eqn. (11) as

\[
\Psi + \dot{q} = \tau \tag{13}
\]

Note that we write \( \Psi \) instead of \( \Psi(t) \) for ease of notation. The control law is given as

\[
\tau = \hat{\Psi} + \dot{q} - K_pe - K_d\dot{e} \tag{14}
\]

where \( \hat{\Psi} \in \mathbb{R}^n \) is the estimate of \( \Psi \), \( K_p \in \mathbb{R}^{n \times n} \) is the tunable proportional gain, \( K_d \in \mathbb{R}^{n \times n} \) is the tunable derivative gain, and \( e = q - q_d \) is the tracking error. Using Eqn. (14), we rewrite Eqn. (13) as

\[
\dot{e} + K_pe + K_d\dot{e} = \hat{\Psi} - \Psi \tag{15}
\]

Using the FAT representation

\[
\Psi = W^T Z(t) + \epsilon(t) \tag{16}
\]

\[
\hat{\Psi} = \hat{W}^T Z(t) \tag{17}
\]

where \( \hat{W}^T \in \mathbb{R}^{n \times n} \) is the estimate of the constant weight matrix \( W^T \), \( Z(t) \in \mathbb{R}^{n} \) is the time-varying vector of basis functions, \( \beta \) is the number of basis functions, and \( \epsilon(t) \) is the approximation error vector, we rewrite Eqn. (15) to get the closed loop dynamics

\[
\dot{e} + K_pe + K_d\dot{e} = -\hat{W}^T Z(t) - \epsilon(t) \tag{18}
\]

where \( \hat{W}^T = W^T - \hat{W}^T \). In the sequel, we will write \( Z \) in place of \( Z(t) \) and \( \epsilon \) in place of \( \epsilon(t) \) for ease of notation. Letting \( \dot{x} = \begin{bmatrix} \epsilon^T & \dot{\epsilon}^T \end{bmatrix}^T \in \mathbb{R}^{2n} \) be the state space vector, we rewrite Eqn. (18) as

\[
\dot{x} = Ax - B \left( \hat{W}^T Z + \epsilon \right) \tag{19}
\]

where

\[
A = \begin{bmatrix} 0 & I_n \\ -K_p & -K_d \end{bmatrix} \in \mathbb{R}^{2n \times 2n}, \quad B = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \in \mathbb{R}^{2n \times n}
\]

Since \( A \) is Hurwitz, we define positive definite matrices \( P \in \mathbb{R}^{2n \times 2n} \) and \( Q \in \mathbb{R}^{2n \times 2n} \) such that \( P = P^T, \ Q = Q^T, \) and \( A^TP + PA = -Q \). The estimate \( \hat{W} \) is obtained by the update law

\[
\hat{W} = -\gamma^{-1} \left( Zx^TPB + \sigma \hat{W} \right) \tag{20}
\]

where \( \gamma \in \mathbb{R}^{n \times n} \) is a tunable diagonal matrix with positive diagonal entries, and \( \sigma \) is a positive tunable scalar.
Theorem 2. The uniform ultimate boundedness of $x$ is guaranteed when the update law of Eqn. (20) is used.

Proof. See [20].

3 New Controller Design

In this section, we develop a new adaptive regressor-free controller that takes advantage of the passivity property of robot dynamics. We call this the adaptive passivity function approximation technique (APFAT) controller in the sequel. We write the unknown time-varying vector $\Psi(t) \in \mathbb{R}^n$ as

$$\Psi = Y\theta$$

(21)

Using Eqn. (21), we rewrite Eqn. (9) as

$$Dr + Cr + \Psi = \tau$$

(22)

Using the FAT representation of Eqn. (16), Eqn. (22) becomes

$$Dr + Cr + \bar{W}^T Z + \varepsilon = \tau$$

(23)

Using the FAT representation of Eqn. (17), the following candidate control law is chosen:

$$\tau = \bar{W}^T Z - Kr$$

(24)

Substituting Eqn. (24) into Eqn. (23) gives the closed loop dynamics

$$Dr = -Cr - Kr - \bar{W}^T Z - \varepsilon$$

(25)

where $\bar{W} = W - \bar{W}$. Consider the candidate update law

$$\bar{W} = -Q^{-1}(Z \tau + \sigma \bar{W})$$

(26)

where $Q \in \mathbb{R}^{n \beta \times n \beta}$ is a tunable diagonal matrix with positive diagonal entries.

Theorem 3. Using the update law of Eqn. (26), the closed loop dynamics of Eqn. (25) is uniformly ultimately bounded.

Proof. First, we introduce the following lemmas that will aid in the stability analysis.

**Lemma 1.**

$$-r^T Kr - r^T \varepsilon \leq \frac{1}{2} \left( \lambda_{\min}(K) ||r||^2 - \frac{4||\varepsilon||}{\lambda_{\min}(K)} \right)$$

(27)

Proof. See Appendix A.

**Lemma 2.**

$$Tr(\bar{W}^T \bar{W}) \leq \frac{1}{2} Tr(W^T W) - \frac{1}{2} Tr(\bar{W}^T \bar{W})$$

(28)

Proof. See Appendix B.

Now, continuing the proof of Theorem 3, we consider the candidate Lyapunov function

$$V = \frac{1}{2} r^T Dr + \frac{1}{2} Tr(\bar{W}^T Q \bar{W})$$

(29)

Taking the time derivative of Eqn. (29) gives

$$\dot{V} = \frac{1}{2} r^T Dr + r^T Dr + Tr(\bar{W}^T Q \dot{\bar{W}})$$

(30)

Evaluating Eqn. (30) along the closed loop trajectory of Eqn. (25) and using Property 2 gives

$$\dot{V} = -r^T Kr - r^T \varepsilon + Tr(\bar{W}^T Q \bar{W})$$

(31)

Using the cyclic property of the trace operation and the fact that $\bar{W} = -\bar{W}$, Eqn. (31) becomes

$$\dot{V} = -r^T Kr - r^T \varepsilon + Tr(\bar{W}^T [-Q \bar{W} - Z \tau])$$

(32)

Using the update law of Eqn. (26), Eqn. (32) reduces to

$$\dot{V} = -r^T Kr - r^T \varepsilon + \sigma Tr(\bar{W}^T \bar{W})$$

(33)

Using Lemmas 1 and 2 and the relationship

$$V \leq \frac{1}{2} \lambda_{\max}(D) ||r||^2 + \frac{1}{2} \lambda_{\max}(Q) Tr(\bar{W}^T \bar{W})$$

(34)

we can write

$$\dot{V} \leq -\alpha V - \frac{1}{2} \left[ \lambda_{\min}(K) ||r||^2 - \frac{||\varepsilon||^2}{\lambda_{\min}(K)} \right]$$

$$+ \frac{1}{2} \left( \sigma Tr(W^T W) - \sigma Tr(\bar{W}^T \bar{W}) \right)$$

$$+ \frac{\alpha}{2} \lambda_{\max}(D) ||r||^2 + \frac{\alpha}{2} \lambda_{\max}(Q) Tr(\bar{W}^T \bar{W})$$

(35)
### Table 1. Summary of Controllers.

<table>
<thead>
<tr>
<th>Controller</th>
<th>AP Controller</th>
<th>FAT Controller</th>
<th>APFAT Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equation</strong></td>
<td>( Y \dot{\theta} - Kr )</td>
<td>( \tau = \hat{W}^T Z + \dot{q}_d - K_p \dot{e} - K_d \dot{e} )</td>
<td>( \tau = \hat{W}^T Z - Kr )</td>
</tr>
<tr>
<td><strong>Equation</strong></td>
<td>[Eqn. (8)]</td>
<td>[Eqn. (14)]</td>
<td>[Eqn. (24)]</td>
</tr>
<tr>
<td><strong>Update Law</strong></td>
<td>( \dot{\theta} = -\Gamma^{-1} Y^T r )</td>
<td>( \hat{W} = -\gamma^{-1} L^T P b + \sigma \hat{W} )</td>
<td>( \hat{W} = -Q^{-1} L^T p + \sigma \hat{W} )</td>
</tr>
<tr>
<td><strong>Equation</strong></td>
<td>[Eqn. (10)]</td>
<td>[Eqn. (20)]</td>
<td>[Eqn. (26)]</td>
</tr>
<tr>
<td><strong>Regressor-Based</strong></td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Slotine and Li's Modification</strong></td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Passivity-Based</strong></td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Tuning Parameters</strong></td>
<td>( K, \Lambda, \Gamma )</td>
<td>( K_p, K_d, \gamma, \sigma, P )</td>
<td>( K, \Lambda, \varphi, \sigma )</td>
</tr>
<tr>
<td><strong>Number of FAT Estimates for an n-DOF Robot</strong></td>
<td>N/A</td>
<td>( n )</td>
<td>( n )</td>
</tr>
</tbody>
</table>

Rearranging terms in Eqn. (35) gives

\[
V \leq -\alpha V + \frac{1}{2} \left[ \alpha \lambda_{\text{max}}(D) - \lambda_{\text{min}}(K) \right] \| r \|^2 + \frac{1}{2} \left[ \alpha \lambda_{\text{max}}(Q) - \lambda_{\text{min}}(Q) \right] \text{Tr} \left( \hat{W}^T \hat{W} \right) + \frac{1}{2} \sigma \text{Tr} \left( W^T W \right) + \frac{\| e \|^2}{2 \lambda_{\text{min}}(K)}
\] (36)

Selecting \( \alpha \leq \min \left\{ \frac{\lambda_{\text{min}}(K)}{\lambda_{\text{max}}(D)}, \frac{\sigma}{\lambda_{\text{max}}(Q)} \right\} \), Eqn. (36) reduces to

\[
V \leq -\alpha V + \frac{1}{2} \sigma \text{Tr} \left( W^T W \right) + \frac{\| e \|^2}{2 \lambda_{\text{min}}(K)}
\] (37)

which implies that \( V < 0 \) if

\[
V > \frac{1}{2 \alpha} \sigma \text{Tr} \left( W^T W \right) + \frac{\| e \|^2}{2 \lambda_{\text{min}}(K)}
\] (38)

This implies uniform ultimate boundedness of \( r \) and \( \hat{W} \). \( \blacksquare \)

The controllers reviewed in Section 2 and the APFAT controller designed in Section 3 are summarized in Tab. 1 to illustrate the control design improvement.

### 4 Simulation

In this section, we simulate the APFAT controller on a robotic system to test its effectiveness. We note that during the simulation, the APFAT controller does not use the robot’s dynamic equation. We also note that the APFAT controller and its update law are shown in Eqns. (24) and (26) respectively.

#### 4.1 Robot Model

The robot model used for this simulation is a six-DOF PUMA500 robot. However, we only use the robot’s first three DOFs \( q_1, q_2, \) and \( q_3 \) as shown in Fig. 1. The link parameters characterize the PUMA500 robot at the Cleveland State University Controls, Robotics, and Mechatronics Laboratory. The actual robot dynamics derived from the Euler-Lagrange method is used to simulate the plant, although the controller has no model information.

For the simulation, we use input constants that convert the robot control signals from \( \text{Nm} \) to \( \text{V} \). The input constants capture the combined effect of gear ratios, amplifier gains, and motor torque constants. The input constants are 0.0543 \( \text{VNm} \), 0.0806 \( \text{VNm} \), and 0.1078 \( \text{VNm} \) for \( q_1, q_2, \) and \( q_3 \) respectively.

#### 4.2 Controller Parameters

We tune the APFAT controller by selecting \( \sigma = 0.009 \). The initial weight matrix \( W \) and vector of basis functions \( Z \) were selected arbitrarily. The initial weight matrix and basis functions were

\[
\hat{w}_i(0) = 0 \in \mathbb{R}^\beta
\]

\[
z_i = \begin{bmatrix}
\cos(w_2t) \sin(w_3t) & \cos(w_2t) \sin(w_3t) & \cdots & \cos(w_2t) \sin(w_3t)
\end{bmatrix}^T \in \mathbb{R}^\beta
\]
for \( i \in [1, 3] \) where \( \omega_Y = \frac{2N_\beta}{T} \). Therefore, \( \hat{\mathbf{W}} = \text{diag}(\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3) \in \mathbb{R}^{3 \times 3} \) and \( \mathbf{Z} = [\mathbf{T}^T \mathbf{z}_2^T \mathbf{z}_3^T]^T \in \mathbb{R}^{3 \beta} \). We chose \( T = \pi \), and the update law gain as \( \mathbf{Q} = 300 \mathbf{I}_\beta \). The controller gains were chosen as \( \mathbf{K} = \text{diag}(\{30 300 30\}) \) and \( \Lambda = \text{diag}(\{6 35 6\}) \). The number of basis functions was chosen as \( \beta = 30 \) to ensure good performance of the controller.

### 4.3 Simulation Results

The performance of the APFAT controller is verified by simulating the robotic system with a time-varying reference trajectory under good and poor initial conditions, without changing the controller gains. The time-varying reference trajectories were selected as \( q_{1d} = \sin(2t) \), \( q_{2d} = 0.25 \sin(2t) \), and \( q_{3d} = 0.5 \sin(2t) - \frac{T}{2} \).

The tracking performance and control signals when good initial conditions were used are shown in Fig. 2. We see that the controller gave good reference trajectory tracking for all the robot joints. The root-mean-square (RMS) errors were 0.013 rad, 0.004 rad, and 0.004 rad for \( q_1 \), \( q_2 \), and \( q_3 \) respectively. We see that despite the larger control signals during the transient phase, the steady-state control signal magnitudes are reasonable and are characterized by no chattering.

Figure 3 shows a comparison of the estimate of the unknown time-varying vector \( \Psi \) with its actual value when good initial conditions were used. We note that \( \Psi \) is time-varying due to the time-varying nature of the reference trajectories. We see that the estimates of the elements of \( \Psi \) are bounded and converge to their true values, although the estimates are characterized by some fluctuations from the true values. Better convergence of estimates with fewer fluctuations can be achieved by increasing the controller gains. However, this can induce unwanted control signal chattering. In this simulation, a trade-off between accurate reference trajectory tracking and good convergence of the estimates, while keeping control signal chattering at a minimum.

The tracking performance of the controller and the control signals when poor initial conditions were used are shown in Fig. 4. The controller still gave good tracking accuracy with
The estimates of the elements of $\Psi$ and their actual values when the APFAT controller was implemented with no initial tracking errors.

FIGURE 3

The estimates of the elements of $\Psi$ and their actual values when the APFAT controller was implemented with nonzero initial tracking errors.

FIGURE 5

The trajectories for $q_1$, $q_2$, and $q_3$ and their respective control signals $u_1$, $u_2$, and $u_3$ when the APFAT controller was implemented with nonzero initial tracking errors.

FIGURE 4

in Fig. 2. The estimates of the elements of $\Psi$ also converge to their true values and remain bounded as seen in Fig. 5.

5 Conclusion

For systems with many degrees of freedom, problems with the computational effort of FAT control can be encountered because of the large matrices that are involved, especially when many basis functions are used. This paper developed a new FAT controller, which simplifies previous adaptive FAT controllers by eliminating the need for simultaneous approximation of a robot’s inertia matrix, Coriolis matrix, and gravity vector. The simplicity in controller design is achieved by approximating the combined product of the regressor matrix and the parameter vector, which is treated as an unknown time-varying function. An adaptive control approach was used to guarantee the stability of the APFAT controller via Lyapunov functions.

The feasibility of the APFAT controller was verified on a three-DOF PUMA500 robot via computer simulations. The simulations showed good trajectory tracking and reasonable control signal magnitudes with both good and poor initial conditions. Future work will include validation of the APFAT controller via experimental tests. For improved performance, the controller gains could be tuned offline or online by an evolutionary optimization algorithm such as particle swarm optimization (PSO) or biogeography-based optimization (BBO). Also, it was assumed in this paper that the robot was not influenced by external forces.
This assumption could be relaxed by generalizing the controller using force estimation or impedance control.

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REFERENCES


A Proof of Lemma 1

In this section, we derive the proof for Lemma 1 from the following equation

\[-r^T K r - r^T \epsilon \leq -\lambda_{\min}(K) \|r\|^2 + \|r\| \|\epsilon\| \quad (39)\]
Let the R.H.S of Eqn. (39) be $E = -\lambda_{\text{min}}(K)\|r\|^2 + \|\varepsilon\|$. By expanding $E$, we get

$$E = -\left[ \frac{1}{2} \lambda_{\text{min}}(K)\|r\|^2 - \left\| \frac{\varepsilon}{\sqrt{2} \lambda_{\text{min}}(K)} \right\| \right]^2$$

$$\leq -\left[ \frac{1}{2} \lambda_{\text{min}}(K)\|r\|^2 - \left\| \frac{\varepsilon}{2\lambda_{\text{min}}(K)} \right\|^2 \right]$$

(40)

Therefore,

$$-r^T K r - r^T \varepsilon \leq -\frac{1}{2} \left[ \lambda_{\text{min}}(K)\|r\|^2 - \left\| \frac{\varepsilon}{\lambda_{\text{min}}(K)} \right\|^2 \right]$$

(41)

\[ \blacksquare \]

B Proof of Lemma 2

In this section, we derive the proof for Lemma 2. Using the fact that $\tilde{W} = W - \bar{W}$,

$$\text{Tr} (\tilde{W}^T \tilde{W}) = \text{Tr} (\tilde{W}^T [W - \bar{W}])$$

$$= \text{Tr} (\tilde{W}^T W) - \text{Tr} (\bar{W}^T \tilde{W})$$

$$\leq \sum_{i=1}^{n} (\|\tilde{w}_i\| \|w_i\| - \|\tilde{w}_i\|^2)$$

$$= \frac{1}{2} \sum_{i=1}^{n} (\|w_i\|^2 - \|\tilde{w}_i\|^2 + \|\tilde{w}_i\| - \|w_i\|)^2$$

$$\leq \frac{1}{2} \sum_{i=1}^{n} (\|w_i\|^2 - \|\tilde{w}_i\|^2)$$

(42)

Therefore,

$$\text{Tr} (\tilde{W}^T \tilde{W}) \leq \frac{1}{2} \left[ \text{Tr} (W^T W) - \text{Tr} (\bar{W}^T \bar{W}) \right]$$

(43)

\[ \blacksquare \]