$H_\infty$ filtering with inequality constraints for aircraft turbofan health estimation

Dan Simon
Cleveland State University
IEEE CDC
December 14, 2006

Supported by the NASA Glenn Research Center, Cleveland, Ohio
Outline

• $H_\infty$ filtering
  – Unconstrained
  – State equality constraints
  – State inequality constraints
• Turbofan engine health estimation
• Simulation results
H\(_\infty\) filtering

- Various approaches have been proposed
- Yaesh and Shaked’s approach (1992)

\[
x_{k+1} = Ax_k + B\nu_k + \delta_k \\
y_k = Cx_k + m_k
\]

\(\nu_k\) and \(m_k\) are uncorrelated, zero mean, white, unity variance
\(\delta_k\) is an adversary’s input (non-random)
\( H_\infty \) filtering

• Use the following predictor/corrector structure:

\[
\hat{x}_{k+1} = A\hat{x}_k + K_k (y_k - C\hat{x}_k) \\
\delta_k = L_k [G_k (x_k - \hat{x}_k) + n_k]
\]

• \( G_k \) is a specified matrix (tuning parameter)
• \( L_k \) is the adversary’s gain matrix
• \( n_k \) is zero mean, white, unity variance, uncorrelated with \( w_k \) and \( m_k \)
$H_\infty$ filtering

• Filtering goal:

$$\sup_{w,m} \frac{\|e\|_G}{\|w\|_2 + \|m\|_2} < 1$$

The solution of a certain saddle point problem ensures that this bound is satisfied.
\( \mathcal{H}_\infty \) filtering solution

\[
K_k = AP_k C^T \quad \hat{x}_{k+1} = A\hat{x}_k + K_k(y_k - C\hat{x}_k)
\]

\[
L_k = AP_k G^T \quad \delta_k = L_k[G_k(x_k - \hat{x}_k) + n_k]
\]

\[
Q_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]
\]

\[
P_k = (I - Q_k G_k^T G_k + Q_k C^T C)^{-1} Q_k
\]

\[
Q_{k+1} = AP_k A^T + BB^T
\]

Covariance bound: \( E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \leq Q_k \)
H∞ with equality constraints

\[
\begin{align*}
    x_{k+1} &= Ax_k + Bw_k + \delta_k \\
    y_k &= Cx_k + m_k \\
    Dx_k &= d \rightarrow D\tilde{x}_k = d \\
    DD^T &= I \\
    V &= D^T D
\end{align*}
\]

Same problem statement as before.
$H_\infty$ with equality constraints

\[
\tilde{K}_k = (I - V)A\tilde{P}_k CT, \quad \tilde{x}_{k+1} = A\tilde{x}_k + \tilde{K}_k (y_k - C\tilde{x}_k)
\]

\[
\tilde{L}_k = (I - V)A\tilde{P}_k GT, \quad \delta_k = \tilde{L}_k [G_k (x_k - \tilde{x}_k) + n_k]
\]

\[
\tilde{Q}_0 = E[(x_0 - \tilde{x}_0)(x_0 - \tilde{x}_0)^T]
\]

\[
\tilde{P}_k = (I - \tilde{Q}_k G_k^T G_k + \tilde{Q}_k C^T C)^{-1}\tilde{Q}_k
\]

\[
\tilde{Q}_{k+1} = (I - V)A\tilde{P}_k A^T (I - V) + BB^T
\]

Covariance bound: $E[(x_k - \tilde{x}_k)(x_k - \tilde{x}_k)^T] \leq \tilde{Q}_k$
Turbofan health estimation
Turbofan health estimation
Turbofan health estimation

Benefits of health estimation:

• Intelligent maintenance scheduling
• Better closed loop control
Turbofan health estimation

DIGTEM: 

\[ x_{k+1} = f(x_k, u_k, p_k, w_k) \]
\[ y_k = h(x_k, p_k, v_k) \]

\( x \) = state vector (16 elements)
\( u \) = control input vector (6 elements)
\( p \) = health parameter vector (8 elements)
\( y \) = measurement vector (12 elements)
\( w \) = process noise
\( v \) = measurement noise
Turbofan health estimation

Augment the state vector with the health parameter vector, and linearize:

\[ x' \leftarrow \begin{bmatrix} x \\ p \end{bmatrix} \]

\[ x_{k+1} = Ax_k + w_k = (16 + 8) \text{ element vector} \]

\[ y_k = Cx_k + v_k \]
Health parameter constraints

![Graph showing expected health parameter degradation with constraints for flight m and flight (m+1).]
## Simulation results

<table>
<thead>
<tr>
<th>Health Parameter</th>
<th>Unconstr Kalman</th>
<th>Constr Kalman</th>
<th>Unconstr $H_\infty$</th>
<th>Constr $H_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan flow</td>
<td>0.192</td>
<td><strong>0.154</strong></td>
<td>0.274</td>
<td>0.232</td>
</tr>
<tr>
<td>Fan eff.</td>
<td>0.164</td>
<td><strong>0.145</strong></td>
<td>0.366</td>
<td>0.261</td>
</tr>
<tr>
<td>Comp. flow</td>
<td>0.194</td>
<td><strong>0.183</strong></td>
<td>0.251</td>
<td>0.220</td>
</tr>
<tr>
<td>Comp. eff.</td>
<td>0.124</td>
<td><strong>0.101</strong></td>
<td>0.168</td>
<td>0.123</td>
</tr>
<tr>
<td>HPT flow</td>
<td>0.167</td>
<td><strong>0.155</strong></td>
<td>0.170</td>
<td>0.145</td>
</tr>
<tr>
<td>HPT enthalpy</td>
<td>0.126</td>
<td><strong>0.113</strong></td>
<td>0.159</td>
<td>0.125</td>
</tr>
<tr>
<td>LPT flow</td>
<td>0.146</td>
<td><strong>0.123</strong></td>
<td>0.169</td>
<td>0.137</td>
</tr>
<tr>
<td>LPT enthalpy</td>
<td>0.249</td>
<td><strong>0.220</strong></td>
<td>0.304</td>
<td>0.235</td>
</tr>
<tr>
<td>Average</td>
<td>0.170</td>
<td><strong>0.149</strong></td>
<td>0.233</td>
<td>0.185</td>
</tr>
</tbody>
</table>

Ave RMS est errors under nominal conditions (500 flights)
Simulation results

<table>
<thead>
<tr>
<th>Health Parameter</th>
<th>Unconstr Kalman</th>
<th>Constr Kalman</th>
<th>Unconstr $H_\infty$</th>
<th>Constr $H_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan flow</td>
<td>1.489</td>
<td>1.214</td>
<td>2.006</td>
<td>0.578</td>
</tr>
<tr>
<td>Fan eff.</td>
<td>2.716</td>
<td>2.551</td>
<td>0.988</td>
<td>0.864</td>
</tr>
<tr>
<td>Comp. flow</td>
<td>1.405</td>
<td>1.276</td>
<td>1.871</td>
<td>1.580</td>
</tr>
<tr>
<td>Comp. eff.</td>
<td>1.200</td>
<td>1.119</td>
<td>1.228</td>
<td>0.865</td>
</tr>
<tr>
<td>HPT flow</td>
<td>1.512</td>
<td>1.318</td>
<td>1.778</td>
<td>1.684</td>
</tr>
<tr>
<td>HPT enthalpy</td>
<td>0.990</td>
<td>0.998</td>
<td>0.539</td>
<td>0.544</td>
</tr>
<tr>
<td>LPT flow</td>
<td>1.000</td>
<td>0.853</td>
<td>1.469</td>
<td>0.865</td>
</tr>
<tr>
<td>LPT enthalpy</td>
<td>2.436</td>
<td>2.218</td>
<td>0.702</td>
<td>0.574</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.716</td>
<td>2.551</td>
<td>2.006</td>
<td>1.684</td>
</tr>
</tbody>
</table>

Max RMS est errors with measurement bias $\pm \sigma/2$ (500 flights)
Conclusion

• Kalman filtering gives better RMS performance, $H_\infty$ filtering gives better worst-case performance under off-nominal operating conditions
• Constrained filtering gives better performance than unconstrained filtering
• Future work: Moving horizon estimation with an $H_\infty$ performance function