GPS Receiver Design for Integrated Navigation Systems

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Biographies

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Abstract

There is much interest in integrated navigation using the Global Positioning System and Inertial Measurement Units (accelerometers and gyroscopes). The results reported in this paper quantify integrated navigation performance as a function of GPS receiver design parameters. The specific application considered is missile navigation.

1 Introduction

To the authors' knowledge, no work has been published on the derivation of GPS receiver design constraints based on integrated navigation requirements. This may be due to the fact that the characterization of GPS/IMU performance is a complex, multidimensional problem which does not lend itself to straightforward analysis. It is much easier to use heuristic calculations than it is to perform rigorous analyses and simulations. These heuristic methods often prove to be adequate for GPS receiver requirement specifications, but sometimes give misleading results. As a result, integrated navigation systems may use GPS receivers which are either underdesigned or overdesigned. That is, the receivers may provide either much better or much worse performance than required. There is therefore a need for improved system-level GPS receiver design methods, so that integrated navigation can be performed more intelligently and cost-effectively.

This paper presents results showing the effect of various GPS user segment parameters on integrated navigation performance. These parameters include the accuracy of the antenna phase response, single versus dual frequency processing, and filter order. The particular application considered is missile navigation. GPS performance can be improved if receiver complexity is increased. So the results in this paper can be used to determine which receiver design is well-suited to a user's particular needs.

Section 2 provides a brief introduction to integrated GPS/IMU navigation, and Section 3 discusses GPS error sources. Section 4 presents some simulation results, and Section 5 contains concluding remarks.

2 Integrated Navigation

Most IMUs have relatively good short term accuracy – that is, they have very little high frequency error. On the other hand, GPS has relatively good long term accuracy – that is, it has very little low frequency error. This combination makes Kalman filter-based integrated GPS/IMU navigation a natural application of GPS, giving a user the best of both worlds [5, 10, 8, 18].

Several fundamental choices must be made when designing an integrated navigation system. First of all, a choice must be made between centralized or decentralized filtering [21]. The use of a single Kalman filter is denoted "centralized filtering," and the use of multiple Kalman filters is denoted "decentralized filtering."

If a user chooses centralized filtering, a choice must be made between embedded (also called integrated or tightly coupled) filtering or nonembedded (also called federated or loosely coupled) filtering. Nonembedded filtering is accomplished by combining the position and velocity solution of a GPS receiver with that of an IMU [19]. However, more accurate and robust solutions are obtained with embedded filtering [11]. With embedded filtering, the Kalman filtering is performed further upstream – that is, the IMU solution is combined with GPS-indicated range and delta range information without the GPS receiver directly computing a solution for position and velocity [6].

In this paper, centralized and embedded filtering is assumed. Due to throughput constraints on the flight computer, the filter may be reduced order. A full order filter would contain over 150 states (IMU and GPS error terms), which may be too cumbersome for real time implementation. In a reduced order filter, only the most significant IMU and GPS errors are used as states in the Kalman filter. This issue is discussed further in Section 4.

Performance can be quantified by Position and Velocity Spherical Error Probable (PSEP and VSEP). SEP is defined as the 50th percentile probability radius of the navigation solution. That is, there is a 50% probability that the vehicle position (velocity) is within PSEP (VSEP) of the navigation solution.

$$\int_0^{SEP}\int_0^{\pi}\int_0^{2\pi} f(x,y,z)r^2\sin\phi\, d\theta\, d\phi\, dr = 0.5 \quad (1)$$

where (x, y, z) are Cartesian coordinates, (r, ϕ, θ) are polar coordinates, and $f(x, y, z)$ is the probability distribution function of the error of the navigation position (velocity) solution. Since the pdf $f(x, y, z)$ is not known, PSEP and VSEP must be solved by using Monte Carlo simulations.

It is important to note the context in which SEP is being discussed. The GPS-only navigation solution has an SEP. But another SEP can be derived for the integrated GPS/IMU navigation solution. Still another SEP can be derived for the IMU-only solution.

Note: If the IMU and GPS error variances are perfectly known and a full-order Kalman filter is used to obtain the navigation solution, then the covariance of the Kalman filter can be propagated into position and velocity space to obtain a position and velocity covariance matrix [3]. It could then be reasonably assumed that $f(x, y, z)$ is Gaussian, and hence $f(x, y, z)$ would be known. This would allow the determination of SEP without resorting to Monte Carlo simulations. However, if a reduced order filter is used, the covariance indicated by the Kalman filter is not the true covariance, and Monte Carlo simulations must be used to obtain SEP.

3 GPS Errors

GPS errors can be categorized as either space segment errors or user segment errors. The space segment errors are quantities over which the user has little or no control, while the user segment errors are directly affected by the design of the receiver. Some of the errors are constant biases, some of the errors are Gauss-Markov processes [4] with an associated time constant τ, and other errors are modelled as white Gaussian noise. The following subsections discuss the errors in more detail. The variances of the GPS errors are subject to some debate. The values used in this study were derived from the literature [9, 14] and flight test experience [2].

3.1 Space Segment Errors

The space segment errors, summarized in Table I, are those over which the user has little or no control.

The GPS satellite ephemerides (position and velocity) are obtained by the receiver from the navigation message which is modulated onto the carrier and transmitted by the satellites. The accuracy of these ephemerides can be expressed in Height, Long-track, and Cross-track (HLC) coordinates. The H axis is parallel to the line segment connecting the earth center with the satellite, the C axis is parallel to the satellite's angular velocity vector, and the L axis completes the orthogonal frame.

Error	Standard Deviation	Type
Satellite Position (HLC frame)	11, 40, 24 feet	Bias
Satellite Velocity (HLC frame)	0.005 feet/sec 0.003 0.003	Bias
Clock Phase	10 nsec	Bias
Clock Frequency	10^{-6} ppm	Bias

Table I: Residual GPS Space Segment Errors

Although the GPS satellite clocks are atomic, they are still subject to errors. These clock errors are continually checked by the control segment, and at least once per day, a correction signal is uplinked to each satellite for subsequent transmission to the user. So most (but not all) of the clock errors are corrected by the user's receiver before the GPS data is processed.

Note that each of these space segment errors are satellite-specific. So if the GPS receiver is tracking n satellites, there are a total of $8n$ space segment errors.

3.2 User Segment Errors

The user segment errors are those which are directly affected by the design of the GPS receiver. These errors are summarized in Table II.

A GPS receiver measures position by tracking a pseudorandom (PR) binary code transmitted by the satellites. This is typically done with a delay-lock loop (DLL) [16], which operates at a rate of 50 Hz. The GPS satellites transmit two PR codes – a 1.023 Mbit/sec code (CA code) and a 10.23 Mbit/sec code (P code). Due to the shorter bit length (in feet) of the P code, a receiver which tracks the P code can generally achieve a better range measurement accuracy than a receiver which tracks the CA code. However, by using narrow correlator spacing in the DLL, a CA code DLL can attain tracking performance comparable with a P code DLL [20]. The tracking error depends on such things as the vehicle dynamics, the design of the loop filter, and the signal-to-noise ratio (SNR). The DLL tracking error is implemented in the integrated GPS/IMU Kalman filter as measurement noise. The receiver simulation employed in this paper utilizes a 2nd order DLL with a bandwidth of 20 rad/sec and a phase margin of 45°.

A GPS receiver measures velocity by tracking the carrier which is modulated by the PR code. This is done with a phase-locked loop (PLL) [13]. The receiver tracks the phase during the measurement interval, and the delta range can then be determined by taking the phase at the end of the measurement interval and subtracting the phase at the beginning of the interval. The phase measurement is called the doppler range measurement, and the difference between two successive doppler range measurements is called a delta range measurement. Usually the higher frequency L_1 (1.575 GHz) carrier is used to obtain vehicle velocity. The tracking error depends on such things as the vehicle dynamics, the design of the loop filter, and the signal-to-noise ratio (SNR). The receiver simulation employed in this study uses a 50 Hz 3rd order Costas loop [12] with a bandwidth of 2 rad/sec and a phase margin of 45°. The PLL is rate-aided by the IMU so that phase lock is maintained during high vehicle dynamics.

SNR has a strong influence on DLL and PLL tracking errors. SNR is determined in part by antenna gain and receiver noise, so the user does have some influence on SNR. Typical SNRs for GPS receivers are in the range of 30 – 40 dB-Hz [7]. The effect of SNR on DLL and PLL tracking errors for those designs discussed in the above paragraphs was determined by Monte Carlo analyses for the missile trajectory used in this paper (see Section 4). The effect of SNR on the tracking errors is given in Table III.

Atmospheric errors [17] are largest for low elevation angles. These errors, which include ionospheric and tropospheric refraction, can at least partly be controlled by the user segment design. Receivers can be designed to track the L_1 (1.575 GHz) carrier, the L_2 (1.228 GHz) carrier, or both. Tracking both frequencies allows for better correction of ionospheric refraction since ionospheric refraction is proportional to the square of the wavelength. The residual ionospheric refraction which remains uncorrected results in both PR code-derived range errors and carrier phase-derived doppler range errors [15]. In the single frequency case, ionospheric modelling helps reduce the ionospheric refraction error by 50 - 75%. But ionospheric modelling cannot be used to predict the *change* in the ionospheric refraction very accurately; so modelling does not reduce the delta range error due to ionospheric refraction. Tropospheric refraction, due to water vapor and other constituents in the atmosphere, is a maximum of about 20 feet. This error can largely be removed with tropospheric modelling. The residual atmospheric errors are functions of the direction between the missile and each satellite. So each satellite has its own atmospheric errors.

Inaccuracies in the measurement of the antenna phase center relative to the IMU navigation reference point introduces errors when integrated navigation is performed. This error is independent of the satellites, so there is only one antenna phase center error.

The antenna phase response induces errors in the measurement of the carrier phase through three mech-

Error	Standard Deviation	Type
DLL Tracking	2 – 10 feet	White Noise
PLL Tracking	.005 – .02 ft	White Noise
Ionospheric Range	8 feet (dual frequency) 25 feet (single frequency)	Bias
Ionospheric Doppler Range	0.02 feet (dual freuquency) 1.6 feet (single frequency)	Markov ($\tau = 200$ sec)
Tropospheric Correction Scale Factor	0.3	Markov ($\tau = 2000$ sec)
Antenna Phase Center	0.1 feet	Bias
Antenna Phase Response	20 – 80°	Markov ($\tau = 30$ sec)
GPS/IMU clock offset	1 msec	Bias

Table II: Residual GPS User Segment Errors

SNR	RMS PLL Noise	RMS DLL Noise
30 db-Hz	0.00776 feet (4.5°)	8.67 feet
32	0.00712 (4.1°)	6.92
34	0.00669 (3.9°)	5.40
36	0.00633 (3.7°)	4.35
38	0.00609 (3.5°)	3.49
40	0.00596 (3.5°)	2.74

Table III: PLL and DLL Tracking Errors

anisms: error in the phase calibration, vehicle attitude error coupled with the slope of the antenna phase response, and ionospheric refraction correction error. If we assume that

$$\begin{aligned} E(e_1^2) = E(e_2^2) &= \sigma_a^2 \text{ (degrees}^2) \\ E(e_1 e_2) &= 0.5\sigma_a^2 \text{ (degrees}^2) \end{aligned} \quad (2)$$

we obtain

$$E(x_{dra}^2) = \begin{cases} (0.00560\sigma_a)^2 \text{ (dual frequency)} \\ (0.00174\sigma_a)^2 \text{ (single frequency)} \end{cases} \quad (3)$$

Of course, in the single frequency case, even though the antenna phase contribution to the doppler range error decreases, the total ionospheric refraction error increases substantially (see Section 3.3). As with the atmospheric errors, the antenna phase response is a function of the direction between the missile and each satellite. So each satellite has a separate antenna phase response error associated with it. Note that the antenna phase error induces a delta range measurement error, but not a range error.

Finally, the difference between GPS and IMU time results in navigation errors. This is because the GPS ephemerides (which are functions of time) must be known to obtain the GPS-derived range and delta range solution. GPS time is the same for each satellite, so there is only one GPS/IMU clock offset error.

3.3 Relative Contributions of GPS Errors

For the missile trajectory considered in this paper (see Section 4), the relative contributions of the GPS errors were obtained using 100 Monte Carlo simulations. An SNR of 30 dB-Hz was used to drive the PLL and DLL errors (see Table III). Antenna phase error varied from 20° to 80°. Tables IV and V show the relative contributions of the GPS errors to range and range rate errors (based on a one-second delta range time interval). These tables show which GPS errors are really significant. For instance, the antenna lever arm contributes less than 1% of the total range and range rate errors; so it would not be wise to spend alot of effort reducing this error. Also note that the PLL tracking error is insignificant; therefore, a PLL design should concentrate on simply maintaing carrier lock, rather than trying to reduce the tracking error. We also note that if a single frequency receiver is used, the antenna phase error is a minor part of the total range rate error, so it would not be desirable to spend alot of money on a highly accurate antenna. But if a dual frequency receiver is used, the antenna phase error can contribute a major portion of the total range rate error, and so it may be desirable to spend some extra money on a more accurate antenna. The reader may be able to draw other conclusions by examining the numbers in Tables IV and V.

Note: The numbers in Tables IV and V may be misleading, because they represent an attempt to portray GPS performance over the course of an entire flight as one set of numbers. In particular, the tropospheric error is maximum at sea level, and drops to zero after about 100 seconds of flight (when the missile rises

GPS Error Term	RMS Range Error and Percent of Total			
	Single Frequency		Dual Freuqency	
Satellite Position	12.7 feet	16 %	12.7 feet	36 %
Satellite Velocity	0.6	< 1	0.6	< 1
Clock Phase	9.8	9	9.8	22
Clock Frequency	0.1	< 1	0.1	< 1
Troposphere	2.2	< 1	2.2	1
Ionosphere	25.0	61	8.0	14
DLL Tracking (SNR = 30 dB-Hz)	8.7	7	8.7	16
Antenna Lever Arm	0.1	< 1	0.1	< 1
GPS/IMU Clock Offset	7.2	7	7.2	11
Totals	31.9	100	21.3	100

Table IV: Relative Contributions of GPS Errors to User-Equivalent Range Errors

GPS Error Term	RMS Range Rate Error and Percent of Total			
	Single Frequency		Dual Frequency	
Satellite Position	0.00426 feet/sec	< 1 %	0.00426 feet/sec	< 1 %
Satellite Velocity	0.00491	< 1	0.00491	< 1
Clock Frequency	0.00100	< 1	0.00100	< 1
Troposphere	0.08617	18 – 19	0.08617	28 – 52
Ionosphere	0.16000	63 – 65	0.00200	< 1
PLL Tracking (SNR = 30 dB-Hz)	0.00776	< 1	0.00776	< 1
Antenna Lever Arm	0.00350	< 1	0.00350	< 1
GPS/IMU Clock Offset	0.07736	15	0.07736	23 – 42
Antenna Phase (20° – 80°)	0.00868 – 0.03472	0 – 3	0.02831 – 0.11324	6 – 48
Totals	0.198 – 0.201	100	0.120 – 0.163	100

Table V: Relative Contributions of GPS Errors to User-Equivalent Range Rate Errors

above the troposphere). So if a user was interested in navigation accuracy only during Stage 3, tables similar to Tables IV and V could be obtained which reflect GPS performance above the troposphere.

4 Simulation Study

The simulation study is based on a representative missile test trajectory starting from Vandenberg Air Force Base in California, and aimed for the South Pacific. The missile has three solid rocket motors, each of which burn for approximately 60 seconds. The missile has a generic IMU which consists of three accelerometers and three gyros, and contains 76 error terms – 10 initial condition errors, 33 accelerometer errors, and 33 gyro errors [3]. The IMU error model is shown in Table VI. The missile's GPS receiver tracks six satellites and has a PDOP which varies between 1.92 and 1.95. The flight computer derives an integrated GPS/IMU navigation solution once per second.

4.1 Reduced Order Filtering

A full order Kalman filter would contain 164 states – 76 IMU error states, 4 global GPS errors (GPS/IMU clock offset and three components of antenna phase center error), and 6 × 14 satellite-specific errors. It may not be feasible to run such a large filter in real time, so two reduced order filters were designed (a lowest order filter, and a medium order filter). The most significant IMU and GPS errors were used as filter states. The variances of the IMU and GPS errors which were used as filter states were increased proportionately to compensate for the fact that most of the IMU and GPS errors were neglected in the Kalman filter. See Table VII for a list of the errors used in the reduced order filters.

The unmodelled errors can be compensated for not only by increasing the *a priori* variance of the modelled errors, but also by adding additional delta range measurement noise to the filter. Numerical results (not published in this paper) indicated that the measurement noise covariance matrix of the filter should

Error	Number of Terms
IMU Initial Conditions	10
Position (feet)	3
Velocity (feet/sec)	3
Azimuth (arcsec)	1
Level (arcsec)	2
Clock Frequency (ppm)	1
Accelerometers	33
Bias (μg)	3
Scale Factor (ppm)	3
g^2 Errors (ppm/g)	6
Float Cocking (rad/rad)	6
Float Coning (arcsec)	6
Bearing Wobble (arcsec)	6
Nonorthogonality (arcsec)	3
Gyroscopes	33
Bias (deg/hr)	3
Scale Factor (ppm)	3
g Errors (deg/hr/g)	9
g^2 Errors (deg/hr/g^2)	18
Total Number of Errors	76

Table VI: IMU Error Model

be increased by 0.04 feet in the delta range measurement to optimally compensate for the unmodelled errors.

4.2 Simulation Results

Each SEP data point in this study was determined by conducting a Monte Carlo analysis of 100 samples. For each Monte Carlo sample, each IMU and GPS error was generated by a Gaussian random number generator. Then the missile flight described earlier in this section was simulated, along with GPS measurements of range and delta range, and IMU measurements of position and velocity. The GPS-only navigation solution had an error (a function of time) for each Monte Carlo sample. These 100 navigation error time functions were used to statistically compute the GPS-only SEP. Also, during each simulated missile flight, a Kalman filter estimated the GPS and IMU errors, and generated an estimated position and velocity based on the IMU error estimates. The position and velocity errors for the 100 Monte Carlo samples were then used to statistically compute the integrated GPS/IMU navigation SEP as a function of time. Since SEP at payload deployment determines weapon delivery accuracy, SEP at 184 seconds following launch (Stage 3 burnout) was used as a scalar measure of integrated navigation performance.

Each of the GPS errors discussed in Section 3 affect SEP, but antenna phase error is of particular interest. It is important for the user to have a good idea of the required antenna phase accuracy, because highly accurate antennas are very expensive.

It has been shown [1] that weapon delivery accuracy depends more on velocity accuracy than position accuracy. So VSEP is of greater interest than PSEP in missile applications.

One of the basic choices which a user needs to make about a GPS receiver is whether to use a single frequency or a dual frequency receiver. Figure 1 shows the GPS-only PSEP as a function of time for a single frequency and a dual frequency receiver. It is seen that the use of a dual frequency receiver results in an improvement of about 15 feet over a single frequency receiver. It is interesting to note that PSEP increases during the last half of the flight. The cause of this phenomenon is currently under investigation.

Figures 2 and 3 show the GPS-only VSEP as a function of time for a single frequency and a dual frequency receiver. It is seen that for a single-frequency receiver, a four-fold improvement in the antenna phase response results in only a marginal improvement of VSEP. This is because the antenna error is dominated by the ionospheric error. Comparing Figures 2 and 3, it is seen that the use of a dual frequency receiver buys only a small advantage (relative to VSEP) over a single frequency receiver at low altitudes, but it buys a large advantage at high altitudes (later in the flight). This is because of the dominance of tropospheric errors at low altitudes. For a dual frequency receiver at high altitudes, an improvement the antenna can make a major improvement in GPS velocity performance.

Table VIII shows the integrated GPS/IMU PSEP for single and dual frequency, and for the three different filter sizes. It is seen that for a single frequency receiver, filter order does not play a large part in determining PSEP. For a dual frequency receiver, filter order has a larger influence on PSEP.

Figure 4 shows integrated GPS/IMU VSEP at payload deployment as a function of antenna phase error for a dual frequency receiver. Note from the figure that both antenna phase error and filter order have a significant influence on VSEP.

Table IX shows integrated GPS/IMU VSEP at payload deployment for a single frequency receiver. (Recall that antenna phase error has a negligible effect on single frequency receiver performance – see Table V.) We see that filter order has a large influence on VSEP. Comparing Table IX with Figure 4, we see that, as expected, single frequency performance is much worse than dual frequency performance. But also note that a single frequency receiver with a full order filter can

Error	Number of Terms	
	Medium Order Filter	Lowest Order Filter
IMU Initial Conditions	10	7
Position (feet)	3	3
Velocity (feet/sec)	3	–
Azimuth (arcsec)	1	1
Level (arcsec)	2	2
IMU Clock Frequency (ppm)	1	1
Accelerometers	6	3
Bias (μg)	3	–
Scale Factor (ppm)	3	3
Gyroscopes	15	6
Bias (deg/hr)	3	–
Scale Factor (ppm)	3	3
g Errors (deg/hr/g)	9	3
GPS Errors (Single Frequency)	22	19
Ionospheric Range (feet)	6	6
Tropospheric Correction Scale Factor (nd)	6	6
Ionospheric Doppler Range (feet)	6	6
GPS/IMU Clock Offset (msec)	1	1
Lever Arm (feet)	3	–
GPS Errors (Dual Frequency)	22	19
Clock Phase (nsec)	6	6
Tropospheric Correction Scale Factor (nd)	6	6
Antenna Phase Error (degrees)	6	6
GPS/IMU Clock Offset (msec)	1	1
Lever Arm (feet)	3	–
Total Number of Error States	53	35

Table VII: Reduced Order Filter Error States

provide a better VSEP than a dual frequency receiver with a low order filter.

Filter Order	Frequency	
	Single	Dual
Lowest Order	26.9 feet	22.7 feet
Medium Order	26.4	20.1
Full Order	26.3	19.6

Table VIII: Integrated GPS/IMU Position Spherical Error Probable at Payload Deployment

5 Conclusion

The performance measure which is of interest for integrated GPS/IMU navigation is Position and Velocity Spherical Error Probable. Monte Carlo simulation studies were conducted to investigate the effects of GPS user segment design on PSEP and VSEP.

Filter Order	VSEP
Lowest Order	0.1323 feet/sec
Medium Order	0.0818
Full Order	0.0751

Table IX: Integrated GPS/IMU Velocity Spherical Error Probable at Payload Deployment with a Single Frequency Receiver

The specific application considered was missile navigation. A Kalman filter combined the GPS and IMU measurements to form a real-time trajectory estimate. The filter rate was 1 Hz, and six GPS satellites were tracked during the flight. Two reduced order filters were compared with the full order filter.

Results have been presented comparing integrated GPS/IMU performance as a function of antenna phase error, single/dual frequency, and filter order. The results presented in this paper indicate that the most important decision to be made about a GPS receiver is whether to use a single or a dual frequency receiver. Filter order is also seen to be a critical parameter. If a dual frequency receiver is used, the quality of the antenna becomes important. Some of the parameters which are seen to be *un*important include the antenna lever arm error and the signal-to-noise ratio (as long as the PLL and DLL maintain lock).

The approach taken in this paper can also be used to determine suitable GPS receiver designs for applications other than missile navigation. This approach allows the user to specify a GPS receiver design which provides enough accuracy, but which is not overly accurate and expensive.

References

[1] J. Diesel, "Integration of GPS/INS for Maximum Velocity Accuracy," *Navigation*, 34:190-211, Fall 1987.

[2] J. Dougherty, H. El-Sherief and D. Hohman, "The Use of GPS for Evaluating Inertial Measurement Errors," *AIAA Journal of Guidance, Control, and Dynamics*, submitted for review.

[3] J. Dougherty, H. El-Sherief, D. Simon and G. Whitmer, "A Design Approach for a GPS User Segment for Aerospace Vehicles," *American Control Conference*, San Francisco, 1993.

[4] A. Gelb, *Applied Optimal Estimation*, Cambridge, MA: MIT Press, 1974.

[5] M. Hadfield and G. Quasius, "INS and GPS Competitive or Synergistic?" *15th Biennial Guidance Test Symposium*, Holloman Air Force Base, NM, pp. 519-534, Sep. 1991.

[6] R. Hartman, "An Integrated GPS/IRS Design Approach," *Navigation*, 35:121-134, Spring 1988.

[7] W. Hurd, J. Statman and V. Vilnrotter, "High Dynamics GPS Receiver Using Maximum Likelihood Estimation and Frequency Tracking," *IEEE Transactions on Aerospace and Electronic Systems*, AES-23:425-436, July 1987.

[8] *Institute of Navigation Satellite Division 5th International Meeting*, Washington, DC: The Institute of Navigation, Sep. 1992.

[9] P. Janiczek and S. Gilbert (editors), *Global Positioning System Papers*. Washington, DC: The Institute of Navigation, vols 1–3, 1980, 1984, 1986.

[10] M. Kao and D. Eller, "Multiconfiguration Kalman Filter Design for High-Performance GPS Navigation," *IEEE Transactions on Automatic Control*, AC-28:304-314, March 1983.

[11] Z. Lewantowicz and D. Keen, "Opportunities and Challenges in Avioinics Integration, INS/GPS – A Case Study," *21st Joint Services Data Exchange for Guidance, Navigation and Control*, Palm Springs, CA, pp. 141-155, Oct. 1992.

[12] W. Lindsey and M. Simon, *Telecommunication Systems Engineering*, Englewood Cliffs, NJ: Prentice-Hall, 1973.

[13] W. Lindsey and C. Chie, *Phase-Locked Loops*, New York: IEEE Press, 1986.

[14] *Navstar GPS User Equipment*, Los Angeles, CA: US Air Force Space Systems Division, January 1991.

[15] W. Pakula, J. Klobuchar, D. Anderson, and P. Doherty, "Ionospheric Errors at L-Band for Satellite and Re-entry Object Tracking in the Equatorial Anomaly Region," *The Effect of the Ionosphere on Radio Wave Signals and System Performance*, Springfield, VA: Naval Research Labs, May 1990.

[16] J. Spilker, *Digital Communications by Satellite*, Englewood Cliffs, NJ: Prentice-Hall, 1977.

[17] J. Spilker, "GPS Signal Structure and Performance Characteristics," in *Global Positioning System, vol. 1*, ed. by P. Janiczek, pp. 29-54, 1980.

[18] R. Stacey, "A Navigation Reference System (NRS) Using Global Positioning System (GPS) and Transponder Aiding," Masters Thesis, Dayton, OH: Air Force Institute of Technology, March 1991.

[19] D. Tazartes and J. Mark, "Integration of GPS Receivers into Existing Inertial Navigation Systems," *Navigation*, 35:105-119, Spring 1988.

[20] A. Van Dierendonck, P. Fenton, and T. Ford, "Theory and Performance of Narrow Correlator Spacing in a GPS Reciever," *Navigation*, 39:265-283, Fall 1992.

[21] M. Wei and K. Schwarz, "Testing a Decentralized Filter for GPS/INS Integration," *IEEE Position, Location and Navigation Symposium*, pp. 429-435, 1990.

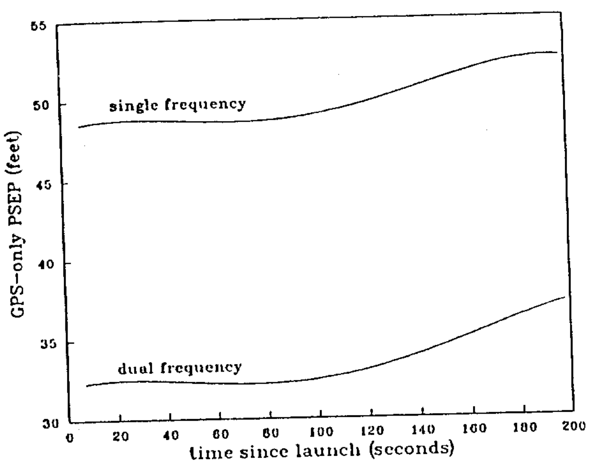

Figure 1: GPS-only Position Spherical Error Probable

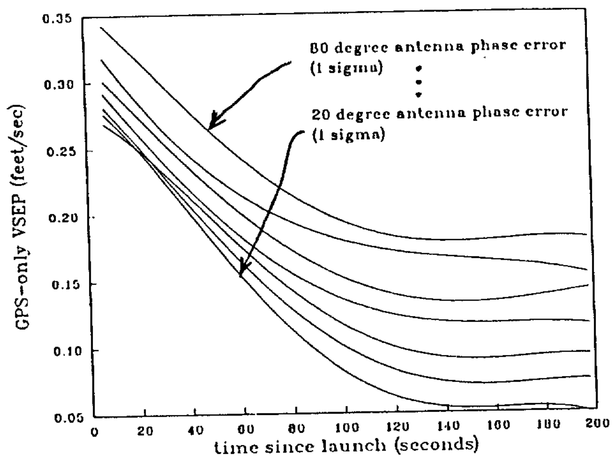

Figure 3: GPS-only Velocity Spherical Error Probable (Dual Frequency)

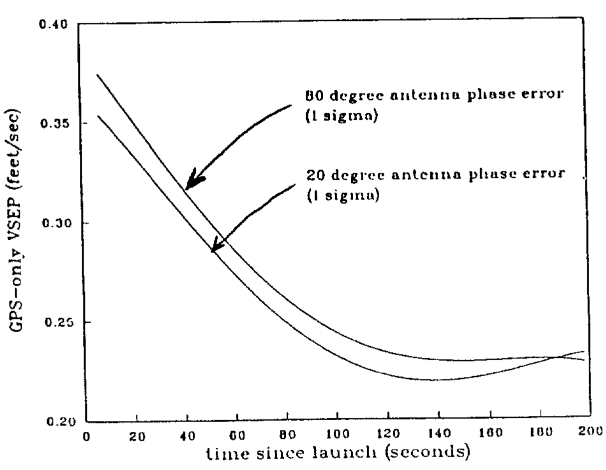

Figure 2: GPS-only Velocity Spherical Error Probable (Single Frequency)

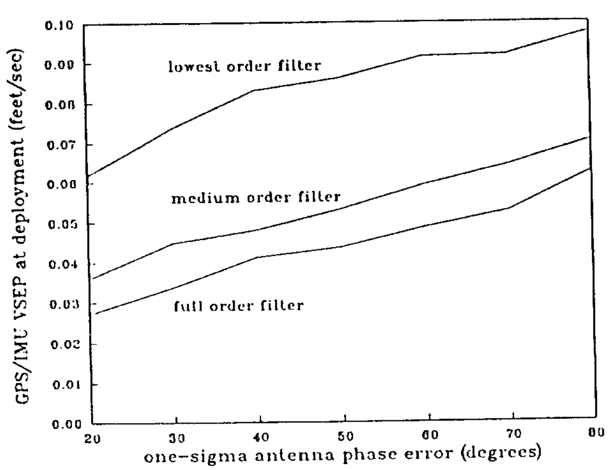

Figure 4: Integrated GPS/IMU Velocity Spherical Error Probable (Dual Frequency)